

**Doctoral Dissertation**

**A Study on the Effectiveness of Integrating GeoGebra Software  
into Teaching and Learning Geometry at Upper Secondary Schools in  
Cambodia**

**SIENG VEASNA**

International Education Development Program  
Graduate School of Humanities and Social Science  
Hiroshima University

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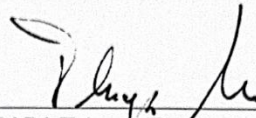
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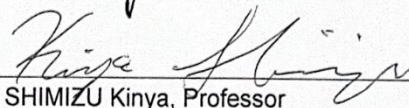
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We hereby recommend that the dissertation by Mr. SIENG VEASNA entitled "A Study on the Effectiveness of Integrating GeoGebra Software into Teaching and Learning Geometry at Upper Secondary Schools in Cambodia" be accepted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN EDUCATION.

Committee on Final Examination:



BABA Takuya, Professor  
Chairperson



SHIMIZU Kinya, Professor



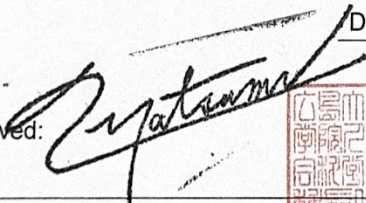
HIRASHIMA Tsukasa, Professor



WATANABE Koji, Professor  
School of Education, Miyazaki International  
University

Date: July 31, 2025

Approved:



MATSUMI Norio, Professor  
Dean



Date: September 4, 2025

Graduate School of Humanities and Social Sciences  
Hiroshima University

## ABSTRACT

Geometry is a crucial domain that enhances the visualization skills required for understanding concepts in school mathematics; however, students often find it difficult to present or manipulate geometric shapes and figures, as well as perform calculation procedures in geometry learning (Mammarella et al., 2017). Scholars have highlighted that students struggle in solving geometry problems through visualization due to their lack of understanding and limited ability to draw and visualise geometry shapes (Hwang et al., 2020; Sari & Slamet, 2018); difficulty in using vocabulary to express mathematical ideas, describe relationships, and create models; and a lack of skills in performing geometric shape representation, which can lead students to have low achievement in solving geometry problems (Gal & Linchevski, 2010; Zhang et al., 2023).

Scholars have noted that students encounter three main challenges in visualisation when solving geometry problems, which include visual perception, visual-spatial skills, and visual representation, each of which is interrelated with the others. For example, to solve a geometric problem, students might first use visual perception to identify the shapes involved. From this experience, students can apply their visual-spatial skills to manipulate these shapes mentally. Finally, visual representations are used to draw diagrams or graphs that aid in a deeper understanding of the problems. These three are implemented in different stages in general education. In primary education, the focus is on visual perception, then shifts to visual-spatial skills in lower secondary education, and finally, to visual representation in upper secondary education (Gal & Linchevski, 2010; Jones & Tzekaki, 2016). While this study targeted upper secondary education, visual representation and conceptual understanding are the focal units of analysis. However, the test items may include some visual perception and visual-spatial skills for further observations. Even though they are not directly addressed, they are interrelated and embedded in the visual representation.

In Cambodia's context, students have low achievement in mathematics, particularly in the geometry domain (MoEYS, 2024), which is highlighted in many documents and reports, such as the Program for International Students Assessment (PISA) in 2022, which shows that only 12% of students achieved level 2 or above in mathematics, while the average for Association of Southeast Asian Nations (ASEAN) is 42.6%, and the Organisation for Economic Co-operation and Development (OECD) is 68.9%, respectively. Moreover, the National Learning Assessment (NLA) conducted

in November 2021 revealed that students' achievement in mathematics remained low compared to the 2016 NLA (UNICEF, 2022), which had an overall score of only 38% (Bhatta et al., 2022). The geometry domain had the lowest scores compared to the other domains (algebra, statistics, measurement, and numbers), with only 35% of grade 6 and 46% of grade 8 students completing the assessment test correctly (Bhatta et al., 2022; MoEYS, 2023). Additionally, the report on the National Examination of grade 12 in the academic year 2020–2021 by the MoEYS showed that only 47.93% of students could solve geometry problems correctly, particularly those related to vector content, which falls under the geometry domain in the mathematics test paper (MoEYS, 2022). These indicators collectively reveal insufficient student achievement in geometry.

Some factors which influence students' low achievement scores include educational policy, the curriculum, school-related issues, personal factors, teachers' knowledge of geometry, the development of effective teaching and learning materials, teaching instruction, integrating dynamic geometry software, curriculum, and social factors (Hasibuan et al., 2019; Kholid, 2022; Purevdorj, 2016; Teodorović, 2011; Ulandari et al., 2019; Wang & Kao, 2022). Among them, integrating dynamic geometry software (DGS) into teaching and learning geometry content, particularly GeoGebra software, is a significant approach to help students overcome difficulties in solving geometry problems through visualization skills, including visual representation, enhancing students' conceptual understanding and improving students' learning achievement (Bantchev, 2010; Chan & Leung, 2014; Cuoco & McCallum, 2018; Hohenwarter et al., 2010; Irshid et al., 2023; Sieng & Thai, 2024b). Along with that, MoEYS developed a guidebook and provided cascade training, including GeoGebra software, for some mathematics teachers to integrate it into mathematics content, especially geometry, in 2020 (MoEYS, 2020b). Nevertheless, the implementation is not fully integrated into schools, such as the mathematics curriculum and textbooks.

Additionally, the curriculum is another influential factor because it is the starting point for teaching and learning activities. Therefore, as the first step, it is important to analyze the curriculum. It should consistently relate to broader educational goals, objectives, domains, subdomains, contents, sub-contents, and learning outcomes. In other words, curriculum coherence is the most dominant predictor for student achievement because it refers to the connection and logical progression of content, skills, and learning outcomes within a curriculum framework,

syllabus, textbook, and assessment (Cuoco & McCallum, 2018; Prawat & Schmidt, 2006; Schmidt et al., 2002; Schmidt & Houang, 2012).

Although specific geometry content was not clearly addressed in the PISA reports and the NLA, the national examination report highlighted that students' achievement is poor in mathematics, specifically in the geometry domain, which refers to vector content (MoEYS, 2022). Moreover, the curriculum analysis also found that vector contents lacked coherence because the sub-contents and learning outcomes did not correspond to each other. Vector contents include geometric shapes and algebra. Students require their knowledge and skills in relation to conceptual understanding and visual representation to be more in-depth in this content. Vector contents are the first aspects covered in the geometry domain of upper secondary schools in the Cambodian mathematics curriculum. However, there is a lack of research and reports investigating the effectiveness of integrating GeoGebra software into teaching and learning geometry and analyzing the Cambodian mathematics curriculum from the perspective of coherence. Therefore, this study analyzed the Cambodian mathematics curriculum with a focus on geometry from the perspective of a coherent curriculum and investigated the integration of GeoGebra software into teaching and learning geometry, focusing on vector content through visual representation and conceptual understanding in solving geometry problems at upper secondary school in Cambodia.

In responding to the research purpose, four research questions have been addressed as follows:

1. What are the students' difficulties in learning geometry?
2. Which approaches does dynamic geometry software use to address the above learning difficulties?
3. What is the current geometry curriculum in upper secondary schools in Cambodia, and which topic is critical?
4. To what extent does a developed coherent mathematics lesson integrating GeoGebra software improve students' geometry learning at upper secondary schools in Cambodia?

The result of this study is to provide the following benefits: (1) they are relevant not only for the Cambodian context but also for other countries where students face the same difficulties in solving geometry problems. (2) They can also serve as a

valuable guide for curriculum developers and textbook writers to consider for future revision. (3) And they can support teachers in preparing teaching and learning materials and promote students' active involvement in mathematics teaching and learning, specifically in geometry.

In responding to the research questions above, the procedures were as follows: the literature review was conducted in response to research questions (1) and (2), the curriculum analysis was conducted in response to research question (3), and the experimental study was done through a field survey in response to research question (4).

For the curriculum analysis, four of the recently revised mathematics syllabi of primary and secondary education in Cambodia, published in 2018, the latest version available during the research period of 2022–2023, and two Singaporean mathematics syllabi published in 2020 were analyzed. All of the documents are considered to be the latest versions. Following the General Topic Trace Mapping (GTTM) procedure (Schmidt et al., 2005b), document analysis was used because it embraces curriculum coherence. The analysis focused on four points: (1) analyzing the arrangement of sub-domains and contents to confirm whether they are repeated or sequenced within and across grades; (2) identifying the relations between contents and sub-contents to see how they fit together at each grade level; (3) identifying the relations between sub-contents and learning outcomes to see how learning outcomes correspond to sub-contents within each grade; and (4) analyzing the contents, sub-contents, and learning outcomes across domains, sub-domains, and grades to see if they correspond and if they are related to confirming the result of the curriculum analysis from the perspective of a coherent curriculum. These four points indicate the importance and necessity of improving the coherence of geometry units in Cambodia's mathematics curriculum. Lastly, the researcher compared the Singapore and the Cambodian mathematics curricula from the perspective of a coherent curriculum to confirm that a coherent curriculum can improve students' learning achievement.

For the field survey, participants involved 168 grade 10 students in Phnom Penh, the capital city of Cambodia. It used a quasi-experimental design with experimental and control groups. The experimental group received instruction with GeoGebra, while the control group followed traditional instruction. The instrument consisted of five test items. Each test item was assessed through visual representation and conceptual understanding. Pre-test and post-test data were analysed using descriptive statistics,

Mann–Whitney U-tests, and Normalised Gain scores to identify statistically significant differences in students' learning achievement before and after the intervention lessons. Twelve students were selected for a semi-structured interview to supplement the students' pre-test and post-test responses. The interview data were analysed by categorizing them according to three levels of student achievement: low, medium, and high. Moreover, the participants' perception of using GeoGebra software in teaching and learning geometry was examined before and after the treatment.

The curriculum analysis results highlighted certain points regarding the lack of coherence in content-relation aspects within the geometry domain. The identified issues include: (1) the relations of the sub-domains and contents are not well organized and are unnecessarily repetitive; (2) contents and sub-contents do not follow a proper order and are not age-appropriate, as some contents are addressed before others that they depend on, and some are beyond the student's grade level; (3) the relations of some contents, sub-contents, and their learning outcomes are not clearly specified, resulting in such problems as a lack of a logical or conceptual connection within and across years; and (4) the relations of domain, sub-domains, contents, sub-contents, and learning outcomes are not clear nor sequenced well as a whole. Compared to the Singapore mathematics curriculum, the comparative analysis reveals that the Singaporean mathematics curricula are highly coherent in terms of design, structure, and content relation, followed by domain, content, sub-contents, and learning outcomes within and across grades. It is well-organized with a logical progression of geometric concepts that correspond with each other. In contrast, the Cambodian mathematics curriculum lacks coherence and has significant gaps in the introduction and development of geometric concepts.

The field survey results revealed the effectiveness of using GeoGebra software in enhancing students' visual representation (VR) and conceptual understanding (CU) of geometry, particularly in solving vector-related problems. The findings suggest that integrating dynamic geometry software, such as GeoGebra, can improve students' VR and CU. The experimental group, which integrated GeoGebra software, showed a marked improvement in VR and CU compared to the control group, as evidenced by higher Normalized Gain scores between the pre-test and post-test. This result suggests that GeoGebra software is not only a supplementary tool but also an effective instructional aid that can significantly enhance students' understanding of complex geometric concepts. Furthermore, as there is a significant improvement between the

pre-test and post-test, based on the students' interviews, they are more interested and confident in improving their knowledge by integrating GeoGebra software into teaching and learning geometry. This study contributes valuable insights into the potential of dynamic geometry software, particularly GeoGebra software, in improving students' mathematical abilities and highlights the need for targeted instructional strategies to support students in overcoming specific learning difficulties in geometry.

However, this study was developed and designed for only two upper secondary schools; the aim was to provide valuable insights into the effectiveness of integrating GeoGebra software in teaching and learning geometry, corresponding to a curriculum that ensures technology integration in lessons. The samples consist only of grade 10 students and two upper secondary schools in Cambodia; therefore, the findings may not be generalised to the entire country. The instrument was developed only for vector content; thus, the other content in the geometry domain, as well as the whole of mathematics, should be investigated. Moreover, the teacher's knowledge and teaching methodology also need to improve.

Considering the results of this study, we recommend that the relevant stakeholders, such as MoEYS, policymakers, curriculum developers, teacher training institutions, and mathematics teachers, consider integrating GeoGebra software into mathematics curricula, textbooks, and teaching methods to improve student learning achievement in mathematics, specifically in geometry.



## TABLE OF CONTENTS

<b>ACKNOWLEDGEMENTS</b> .....	xiv
<b>TABLE OF CONTENTS</b> .....	ii
<b>LIST OF TABLES</b> .....	vii
<b>LIST OF FIGURES</b> .....	x
<b>LIST OF ABBREVIATIONS</b> .....	xi
<b>ABSTRACT</b> .....	iii
<b>CHAPTER 1: INTRODUCTION</b> .....	1
1.1 THE EDUCATION SYSTEM IN CAMBODIA .....	1
1.1.1 Overview of Education in Cambodia .....	1
1.1.2 Curriculum Framework of General and Technical Education .....	3
1.1.3 Cambodian Mathematics Education .....	5
1.1.4 Mathematics Assessment .....	9
1.1.5 The Use of ICT in Mathematics Education .....	14
1.2 PROBLEM STATEMENT .....	16
1.3 RESEARCH OBJECTIVES AND QUESTIONS .....	17
1.4 SIGNIFICANCE AND LIMITATIONS OF THE STUDY .....	18
1.5 STRUCTURE OF THE DISSERTATION .....	19
<b>CHAPTER 2: LITERATURE REVIEW</b> .....	<b>22</b>
2.1 OVERVIEW OF GEOMETRY .....	22
2.1.1 Geometry .....	22
2.1.2 Aspect of Curriculum Coherence .....	27
2.1.3 Conceptual Understanding .....	30
2.2 ASPECTS OF VISUALIZATION IN LEARNING GEOMETRY .....	33
2.2.1 Visualization .....	33

2.2.1.1	Visual Perception .....	34
2.2.1.2	Visual-Spatial Skills .....	35
2.2.1.3	Visual Representation .....	37
2.2.1.4	Relationship Between Visual Perception, Visual-Spatial Skills, and Visual Representation .....	40
2.2.2	Relationship Between CU and VR .....	42
2.2.3	Relationship Between CU and Visualization .....	44
2.3	APPROACHES IN GEOMETRY LEARNING AND DYNAMIC GEOMETRY SOFTWARE .....	47
2.3.1	Approaches in Geometry Learning .....	47
2.3.1.1	Relationship Between DGS and CU .....	48
2.3.1.2	Relationship Between DGS and Curriculum Coherence .....	49
2.3.2	Type and Features of DGS .....	51
2.3.3	Comparison of DGS .....	52
2.3.4	Description of GeoGebra Software .....	54
2.3.5	Features and Strengths of GeoGebra Software .....	57
2.3.6	Global Trends in Integrating GeoGebra Software into Upper Secondary Education .....	60
2.4	SUMMARY AND CONCEPTUAL FRAMEWORK .....	64
	<b>CHAPTER 3: RESEARCH METHODOLOGY</b> .....	67
3.1	METHODOLOGY OF GEOMETRY CURRICULUM ANALYSIS.....	67
3.1.1	Data Source .....	67
3.1.2	Data Analysis Method .....	68
3.2	METHODOLOGY OF FIELD SURVEY .....	69
3.2.1	Research Location and Participants .....	69
3.2.2	Research Design and Procedure.....	71
3.2.3	Research Instrument.....	74

3.2.4	Validation of the Instrument.....	82
3.2.5	Data Collection.....	83
3.2.6	Data Analysis .....	84
3.2.7	Ethical Consideration .....	90
<b>CHAPTER 4: RESULTS</b>	.....	<b>92</b>
4.1	RESULT OF CURRICULUM ANALYSIS.....	92
4.1.1	Arrangement of Domain, Sub-domain, and Contents.....	92
4.1.2	Relation of Sub-Domains and Contents.....	95
4.1.3	Relation of Contents and Sub-contents Across Grades .....	96
4.1.4	Relation Between the Contents, Sub-contents, and Learning Outcomes	98
4.1.5	Relation of the Domain, Sub-domains, Contents, Sub-contents, and Learning Outcomes Across the Grades.....	100
4.1.6	Field Survey Interview with a Focus on Curriculum.....	103
4.2	RESULT OF THE COMPARATIVE ANALYSIS OF THE CURRICULUM .....	108
4.2.1	Constructed Curriculum Structure.....	108
4.2.2	Comparison of Sub-Domains and Contents.....	110
4.2.3	Comparison of Contents, Sub-contents, and Learning Outcomes Between Mathematics Curricula of Both Countries.....	112
4.3	RESULTS OF FIELD SURVEY.....	115
4.3.1	Demographics of the Respondents.....	115
4.3.2	Descriptive Statistics of Pre-Test and Post-Test on VR and CU	116
4.3.3	Students' Improvement in Solving Vector Concepts .....	117
4.3.4	Differentiation between Pre-Test and Post-Test of Students' VR and CU Through Intervention .....	122
4.3.5	Differentiation of Students' VR and CU by Test Items.....	124

4.3.6 Differences Between Control and Experimental Groups Using Normalized Gain Score .....	125
4.3.7 Relationship Between VR and CU .....	127
4.4 STUDENT INTERVIEW RESULTS.....	129
4.4.1 Selected Criteria for Interview.....	129
4.4.2 Students' Improvement Change from Pre-test to Post-test of VR and CU	132
4.4.3 Summary of Students' Interview Results.....	150
4.4.4 Students' Perception of Integrating GeoGebra Software.....	152
CHAPTER 5: FINDING AND DISCUSSION.....	154
CHAPTER 6: CONCLUSION AND RECOMMENDATION .....	159
6.1 Conclusion.....	159
6.2 Recommendation .....	160
6.3 Future Issues.....	161
<b>REFERENCES</b> .....	164
Appendix 1: Description of the Six Levels of Mathematics Proficiency in PISA 2022 .....	179
Appendix 2: A Sample Lesson Plan on Vector Plane and Vector Space..	182
Appendix 3: A Sample Lesson Plan on Vector Space.....	189
Appendix 4: Main Survey Lesson Plans .....	198
Appendix 5: Worksheets .....	229
Appendix 6: Pilot Test .....	235
Appendix 7: Pre- and Post-Test .....	238
Appendix 8: Scoring Rubric .....	241
Appendix 9: Interview Guide for Curriculum Developers .....	257
Appendix 10: Interview Guide for Teachers .....	258
Appendix 11: Interview Guide for Students.....	259

Appendix 12: Research Title .....	260
Appendix 13: Research Ethics Clearance .....	261
Appendix 14: Approval from MoEYS .....	262

## LIST OF TABLES

Table 1. Education systems after independence from France (1953) to the present era.....	2
Table 2. The main concepts of geometry for general and technical education	6
Table 3. Percentage of students by proficiency level in mathematics .....	10
Table 4. The available dynamic software packages are suitable for integration into science and mathematics .....	14
Table 5. Geometry curriculum at upper secondary schools in Cambodia .....	24
Table 6. The percentage of teaching hours allocated for the geometry domain in Cambodian mathematics compared to other domains for the science track. ....	26
Table 7. The percentage of teaching hours allocated for the geometry domain in Cambodian mathematics compared to other domains for the social science track .....	27
Table 8. Aspects of curriculum coherence .....	30
Table 9. Differences and Relations Among Visual Perception, Visual-Spatial Skills, and VR.....	42
Table 10. Classification of the use of DGS in different representations .....	51
Table 11. DGS packages are available online, offline, or both .....	51
Table 12. Comparison of the dynamic geometry software packages used to visualize geometry shapes and algebra.....	53
Table 13. The features of GeoGebra software. ....	58
Table 14. The trends in integrating GeoGebra software by education level ...	59
Table 15. The utilization of GeoGebra software in different countries.....	60
Table 16. Summary of the participants.....	71
Table 17. Summary of the intervention design for control and experimental groups.....	72
Table 18. The implementation period for the field survey .....	75
Table 19. Sample of pre-test and post-test for the field survey.....	78
Table 20. Test items development .....	80
Table 21. The sample of VR and CU expressed in textbook grade 10 .....	81
Table 22. The scoring rubric of Test Item Number 1 .....	84

Table 23. The classification of a relationship (effect size).....	88
Table 24. The categories and percentages of the normalized gain scores ....	88
Table 25. Strength of correlation criteria .....	89
Table 26. Response Coding for the results of the interviewees. ....	89
Table 27. Interviewed students for qualitative analysis.....	90
Table 28. The arrangement of the geometry domain, sub-domains, and contents from grades 1 to 12.....	92
Table 29. The relation between a sub-domain and its contents.....	95
Table 30. The relation between contents and sub-contents within and across grades.....	96
Table 31. The relation of content, sub-contents, and learning outcomes within the grade level. ....	98
Table 32. The corresponding contents, sub-contents and learning outcome of the vector within grade 10 .....	99
Table 33. The relation of a domain, sub-domains, contents, sub-contents, and learning outcomes across the grades. ....	100
Table 34. Geometry contents are allocated in upper secondary education in Cambodia. ....	103
Table 35. Summary of the constructed curriculum structures for Cambodia and Singapore .....	109
Table 36. Example of a comparison of sub-domains and contents.....	110
Table 37. An example of the mathematics curriculum of both countries for grade 1.....	112
Table 38. The number of students who responded .....	115
Table 39. Reliability of the test items using Cronbach's Alpha.....	116
Table 40. The descriptive statistics of students' responses through the pre-test and post-test .....	116
Table 41. Test item 5 and its expected answer .....	119
Table 42. An example of students' responses to test item 3.....	120
Table 43. The comparison of the improvement in test scores between the pre-test and post-test.....	124
Table 44. Normalized Gain Score Percentage (%).....	126

Table 45. The correlation between VR and CU for the pre-test. ....	128
Table 46. Distribution score of the VR and CU pre-test.....	129
Table 47. Distribution of students per class according to the achievement level .....	131
Table 48. Total distribution score of VR and CU through pre-test.....	132
Table 49. Students' improvement scores are categorized by pre-test and post- test of VR and CU. ....	133
Table 50. Students' improvement changes by categories for test item 1. ....	134
Table 51. Low-achievement students' response to the test item 1 from the control group. ....	137
Table 52. The improvement in students' results based on the interview regarding test item 1 for students with low achievement.....	138
Table 53. Low-achievement students' response to the test item 1 from the experimental group .....	140
Table 54. The improvement in students' results based on the interview regarding test item 1 for students with low achievement.....	141
<i>Table 55. students' improvement throughout the pre-test and the post-test of test item 5. ....</i>	143
Table 56. Low-achievement students' response to the test item 5 from the control group.....	145
Table 57. The improvement in students' results based on the interview regarding test item 1 for students with low achievement.....	146
Table 58. Low-achievement students' response to the test item 5 from the experimental group .....	147
Table 59. The improvement in students' results based on the interview regarding test item 5 for students with low achievement in the experimental group. .....	148
Table 60. Summary of the interview results .....	150

## LIST OF FIGURES

Figure 1. The overall percentage of correct answers for the National Learning Assessment in grade 8.....	12
Figure 2. Result of the national examination of grade 12 in the academic year 2021–2022.....	13
Figure 3. Construction of thesis .....	21
Figure 4. An example of the relation between visual perception, visual-spatial skills, and VR in identifying a triangle among other polygons.....	40
Figure 5. Properties of triangles.....	45
Figure 6. Conceptual framework of the relationship between visualization and CU .....	46
Figure 7. GeoGebra interface capture.....	55
Figure 8. The GeoGebra interface presents a VR and enables CU.....	56
Figure 9. The conceptual framework of this study .....	66
Figure 10. A map of the Kingdom of Cambodia showing the research location. ....	69
Figure 11. The research framework and procedure of the intervention lesson .....	73
Figure 12. A sample worksheet.....	77
Figure 13. Pre-test and post-test items of the field survey.....	83
Figure 14. The percentage of students' responses was correct for pre-tests .....	118
Figure 15. Percentage of students who responded with correct answers on VR and CU.....	121
Figure 16. Normal distribution for pre-test and post-test.....	123
Figure 17. The differentiation between the pre-test and post-test on VR & CU in percentage of the experimental group and the control group .....	125
Figure 18. Test item 1.....	136
Figure 19. The test items 5 .....	144

## **LIST OF ABBREVIATIONS**

ADB:	Asian Development Bank
ASEAN:	Association of Southeast Asian Nations
CEC:	Early Childhood Education
CU:	Conceptual Understanding
DCD:	Department of Curriculum Development
DGS:	Dynamic Geometry Software
DIT:	Department of Information and Technology
EQAD:	Educational Quality Assurance Department
ESP:	Education Strategic Plan
GNI:	Gross National Income
GTTM:	General Topics Trace Mapping
ICT:	Information, Communication and Technology
IT:	Information Technology
JICA:	Japan International Cooperation Agency
LSS:	Lower Secondary School
MoEYS:	Ministry of Education, Youth and Sport
NGO:	Non-Governmental Organization
NGS:	New Generation School
NIE:	National Institute of Education
NIEI:	NIE International Pte Ltd
NLA:	National Learning Assessment
NWS:	Network School
OECD:	Organisation for Economic Co-operation and Development

OHS:	Ordinary High Schools
PIRLS:	Progress in International Reading Literacy Study
PISA:	Programme for International Student Assessment
PRI:	Panhchak Research Institute Co., Ltd
RGC:	Royal Government of Cambodia
RTTC:	Regional Teacher Training Center
SRS:	Secondary Resource Schools
SRS:	Upper Secondary School
STEM:	Science, Technology, Engineering, and Mathematics
TIMMS:	Trends in International Mathematics and Science Study
UNICEF:	United Nations Children’s Fund
USESDP 2:	Second Upper Secondary Education Sector Development Program
VR:	Visual Representation
WB:	World Bank

## **DEDICATION**

I would like to dedicate this dissertation to my parents, Mr. SIENG POEUN and Mrs. CHHIM SOKHORN; my father-in-law, Mr. UM SRY; my mother-in-law, Mrs. HUN RIN; my beloved wife, Mrs. SRY MALIS; and my dear children, Mr. POEUN SOVANPANHA and Miss POEUN KEOV PUTTHIMA. Their precious love, encouragement, and patience are immeasurable to me, making me remember them deeply in my heart and soul. I hope that they will be happy and satisfied with my achievement.

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## **CHAPTER 1: INTRODUCTION**

This chapter provides the following background information: an overview of education in Cambodia, the Cambodian curriculum framework, Cambodian mathematics education focusing on the geometry domain, mathematical assessment from international to local assessment, and the integration of information, communication and technology (ICT) into mathematics education. Additionally, this chapter states the problem statements, research objectives, research questions, and significance of the study, and describes the structure of the dissertation.

### **1.1 THE EDUCATION SYSTEM IN CAMBODIA**

#### **1.1.1 Overview of Education in Cambodia**

The Cambodian education system has existed since at least the thirteenth century. Historically, education in Cambodia was conducted in the Wats (Buddhist monasteries) and was available only to the male population. It focused on basic literature, social skills, and practical skills, such as carpentry, artistry, craftwork, construction, and playing musical instruments. This traditional education system was gradually changed during the period in which Cambodia was a French colony (1863–1953) (McNamara, 2022).

Cambodia became independent from France in 1953. Even though the French retreated from Cambodia, the education system during the King Norodom Sihanouk (Kingdom of Cambodia) period still followed the French model (6+4+2+1), which includes six years of primary education, four years of lower secondary education, two years of upper secondary education, for general education; and an additional year for pre-university education.

Primary education was divided into two stages, each comprising three years. Khmer (the official Cambodian language) was the language of teaching in the first stage, but French was used in the second stage. Secondary education was also divided into two stages (lower secondary and upper secondary). It took four years to complete the first stage and three years for the second stage, including pre-university.

The Ministry of Education, Youth, and Sports (MoEYS) strictly inspected and exercised complete control over the entire system, such as establishing syllabi, hiring

and paying teachers, providing supplies, capacity-building teachers, and inspecting schools (Dy, 2004; Neau, 2003).

The current general education practice only started after the end of the Civil War of 1975–1979 (Khmer Rouge). From the end of the Civil War up to the present, the general education system has been structured into three levels of education: primary education, lower secondary education, and upper secondary education. Along with this, MoEYS has reorganized the general education system three times. First, from 1979 to 1986, a ten-year system was followed: four years of primary education, three years of lower secondary education, and three years of upper secondary education (4+3+3). Second, from 1986 to 1996, an eleven-year system was implemented: five years of primary education, three years of lower secondary education, and three years of upper secondary education (5+3+3). From 1996 until today, a twelve-year system has been in place: six years of primary education, three years of lower secondary education, and three years of upper secondary education (6+3+3) to increase teaching and learning hours for deepening children’s knowledge (Naron, 2015; SEM & HEM, 2016). Table 1 shows the period for each of Cambodia’s education systems.

*Table 1. Education systems after independence from France (1953) to the present era*

No	Period	Education system
1	Thirteenth century to 1953	Wat schools
1	King Norodom Shihanouk (1953–1970)	French model (6+4+2+1)
2	Khmer Republic (1970–1975)	French model (6+4+2+1)
3	Khmer Rouge (1975–1979)	No formal education
4	1979–1986	10-year education formula (4+3+3)
5	1986–1996	11-year education formula (5+3+3)
6	1996–Present	12-year education formula (6+3+3)

**Source:** Developed by the author through the literature review.

Even though the Cambodian education system has expanded for general education from one period to another, numerous documents and reports show that student learning is still at a low level (Bhatta et al., 2022; MoEYS, 2019b; UNICEF, 2022).

## 1.1.2 Curriculum Framework of General and Technical Education

The development of the Cambodian curriculum framework seeks to improve the quality of education in response to the changing world and the requirements of the Royal Government of Cambodia's (RGC) Rectangular Strategy Phase IV<sup>1</sup> (RGC, 2018). This strategy aims to help Cambodia achieve an upper-middle-income country by 2030 and a high-income country by 2050<sup>2</sup> by endorsing more reflective and relevant learning to ensure economic growth and to enhance its ability to compete for new employment in national, regional, and international markets. In response to the market demands, ASEAN integration, and the changing world, developing a modern and evidence-based curriculum framework by integrating ICT in education for general education, specifically, the integration of ICT into learning content, is the highest priority to promote Cambodian citizens with high potential (MoEYS, 2016; RGC, 2019).

MoEYS has revised and developed the National Curriculum Framework for General and Technical Education to contribute to human resource development by equipping learners with the knowledge, skills, and attitudes to participate in national development (MoEYS, 2016). The revised curriculum framework aims to develop

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<sup>1</sup> The Rectangular Strategy Phase IV (2018–2023) is a foundational policy document that outlines the RGC's development agenda, emphasizing human resource development as a top priority. It focuses on improving the quality of education, science, and technology; vocational training; public health and nutrition; and gender equity and social protection. Along with this, MoEYS developed the Education Strategic Plan 2019–2023. This plan aligns with the Rectangular Strategy Phase IV. It aims to enhance education quality, expand access, and ensure effective governance. Key initiatives include curriculum reform, teacher training, and the promotion of STEM education.

<sup>2</sup>. According to the World Bank (2024), the different levels of classification are based on Gross National Income (GNI) per capita. It classifies low-income countries as having a GNI per capita of \$1,145 or less; lower-middle-income countries as having in the range of \$1,146–4,515, upper-middle-income countries as having in the range of \$4,516–14,005, and high-income countries as having higher than \$14,005.

students' competencies to meet the level required by 21st-century skills and science, technology, engineering, and mathematics (STEM) education, and to integrate ICT into education. MoEYS stated that acquiring knowledge of STEM and ICT is a basis for further study and daily-life communication (MoEYS, 2016, p. 3).

Integrating ICT into the Cambodian curriculum framework is one of the possible approaches to improve the students' competencies by providing them with the dynamic software and skills necessary to experience a digital education and an interconnected world of education. This integration will prepare students for academic success and equip them with the skills and competencies needed for future careers and responsible citizenship. In alignment with the established curriculum framework, MoEYS has explicitly highlighted in the Education Strategic Plan (ESP: 2019–2023) that 21st-century skills are essential for citizens in a knowledge-based economy (MoEYS, 2019a).

Prior to the ESP, MoEYS developed policies and strategies on ICT in education in 2004 (MoEYS, 2004). The Cambodian government adopted a policy entitled *Policy and Strategies: Information and Communication Technology in Education in Cambodia*. The MoEYS set ambitious goals for the ICT in Education policy, which included increasing access to basic education for all by using ICT as a major tool, improving the quality of general education and creating a workforce eligible to participate in the global knowledge-based economy. They decided to include it in the general education curriculum from grades 4 to 12 (MoEYS, 2018a, 2018b, 2018c), aiming to introduce ICT into the general and technical education curriculum for future revision.

Even though the integration of ICT is necessary to improve the quality of education, the latest curriculum framework has only introduced general ICT usage for administrative support (MoEYS, 2016), such as basic computer literacy, computer usage in daily work, graphic design, sending documents and information via computer systems, and technology in classroom presentations, which means that the integration of ICT into learning content has not yet been fully realized in classroom practice. Moreover, MoYES aims to integrate ICT into content subjects for general education and technical education (K-12), for future curriculum reform (MoEYS, 2025).

### **1.1.3 Cambodian Mathematics Education**

Mathematical knowledge plays a crucial role in comprehending the content of various disciplines, as it fosters logical reasoning and problem-solving skills applicable across different fields (Celik & Ozdemir, 2020). Additionally, integrating mathematics into other content areas has improved students' grasp of mathematical concepts, facilitating interdisciplinary learning (Kaur et al., 2022). A strong foundation in mathematics can enhance a student's ability to learn and understand other subjects (Celik & Ozdemir, 2020; Kaur et al., 2022).

Cambodian mathematics education was developed as a foundation subject for developing students' problem-solving and analytical skills (MoEYS, 2016). As a core subject, mathematics is essential for educational human resource development and enables other skills to meet the standards of the world economy. Moreover, Cambodia is committed to enhancing mathematics as a key enabling discipline that underlines and is at the heart of the country's economic, environmental, and social development (Roath, 2015).

The current development of mathematics education in Cambodia is crucial for enhancing and expanding knowledge in analyzing mathematical concepts, such as logical reasoning, graph explanation, analysis, and abstract thinking.

At the primary level, mathematics education focuses on building fundamental skills. Students are taught basic arithmetic operations and simple problem-solving exercises. The aim is to develop students' numeracy skills and basic mathematical understanding, which are essential for their everyday lives and further education (MoEYS, 2018e).

At the lower secondary level, mathematics education becomes more complex. The emphasis shifts from simple calculation to abstract thinking and logical reasoning. Students begin to tackle more sophisticated problems that require applying mathematical concepts, such as equations and basic geometric principles (MoEYS, 2018d).

At the upper secondary level, mathematics education becomes more specialized; specifically, students at this level need to develop a deeper understanding of the relationship between mathematics and real-world applications. The content is

more abstract and complex. Students explore geometry, algebra, trigonometry, calculus, probability, and statistics in greater depth (MoEYS, 2018f, 2018g; SamAn, 2018).

Although mathematics education in Cambodia is structured across different educational levels, it still faces several challenges, such as an overloaded curriculum, topics that are too difficult for general education students, and content that might not be age-appropriate (NIEI & PRI, 2021). Specifically, the geometry domain presents one of the challenges for Cambodian students, as highlighted in the analysis report on the grade 12 national examination in the year 2021-2022 (MoEYS, 2022), the technical report on the grade 8 national learning assessment in the year 2021-2022 (MoEYS, 2023), and the report on the results of grade 11 students' achievement from the national assessment in the year 2018 (MoEYS, 2019b).

In mathematics, geometry is one of the more complicated domains for teachers to teach and for students to learn, specifically in upper secondary education (NIEI & PRI, 2021). It deals with shapes and algebra; many students struggle to grasp the concepts and interpretations as mathematical expressions. In Cambodian education, geometry is introduced from early to upper secondary education for general education, emphasizing various educational levels (MoEYS, 2018d, 2018e, 2018f, 2018g). Table 2 shows the main focus of geometry content across different levels of Cambodian general and technical education.

*Table 2. The main concepts of geometry for general and technical education*

<b>Primary</b>	<b>Lower Secondary</b>	<b>Upper Secondary</b>
Shapes	Shapes and properties of 2-dimensional and 3-dimensional geometric figures: Students learn about the geometric foundation, geometric shapes in a plane, and geometric shapes in space.	<b>Plane geometry:</b> Students learn about geometry in a plane (coordinates of points in a plane, linear equations, circular equations, and solving inequations by using graphs), vectors in a plane (vectors, operations on vectors, and practice on vectors), and transformation in a plane (moving transformation, symmetric

		transformation, rotate transformation, and homothety transformation).
Properties	Relation between lines, angles, and ratios: Students learn about symmetric figures; the properties of a circle, triangle, and regular polygon; and trigonometric ratios.	<b>Space geometry:</b> Students learn about geometry in space, lines and parallel planes in space, and orthogonals in space.
Show the geometric shape in 2- and 3-dimensional.	Development in the learning progress of (a) plane geometry (2D), such as the construction of geometric shapes that is, their perimeter; the surface of a plane, circle, and triangles; the distance between two points; the equation of a line; and scale and (b) solid geometry (3D), with a focus on the volume and surface of a solid, including those of its shape, and similarities among solids.	Analysis of space geometry: Students learn about vectors in space, the scalar product of two vectors and its applications, plane equations, line equations in space, and sphere equations.
	Proof of Pythagorean and Thales' theorem and similar triangles.	<b>Conic Section:</b> Students learn about parabolas, ellipses, and hyperbolas.

**Source:** The author developed this table based on the detailed Cambodian mathematics syllabus (MoEYS, 2018d, 2018e, 2018f, 2018g).

Table 2 shows the different geometry contents at each level in general education. The geometry contents were structured progressively across three levels: Primary, Lower secondary, and Upper secondary, with each level building upon the previous knowledge to develop students' spatial reasoning, mathematical understanding, and problem-solving skills. In primary education, the contents allocated in the curriculum are to provide knowledge and develop skills regarding the properties of 2D and 3D shapes by using simple 2D and 3D objects (MoEYS, 2018e). A shape refers to the form of an object. It describes geometrical properties, such as length, width, curves, edges, and angles. A shape does not include physical properties, like texture, material, or weight. For example, a circle is a 2D shape defined by all points equidistant from a center; a rectangle has four sides, with opposite sides equal. Meanwhile, an object refers to a physical or abstract thing that exists. The object's shape can describe it, including material, color, weight, function, and purpose. An object can be 2D or 3D and may be concrete (e.g., a book) or abstract (e.g., a mathematical object). An object often has volume and mass in physical contexts. For example, a ball is an object. Its shape is a sphere. Another example is a chair, an object with a complex shape that can be described using geometric components.

In lower secondary education, students' primary education experiences are built upon to add more depth to the properties of 2D and 3D shapes, including the relationships between lines, angles, and ratios. Students learn about symmetric figures and the properties of common geometric shapes such as circles, triangles, and regular polygons. At this level, the geometry curriculum introduces trigonometric ratios, which help them analyze angle relationships more precisely. Additionally, students construct geometric figures, calculate perimeter and area, understand scale, and determine distances between points. They also begin studying solid geometry, focusing on volume, surface area, and similarities among 3D shapes. Logical thinking is further enhanced through the introduction of important geometric theorems, such as the Pythagorean Theorem and Thales' Theorem, and the concept of similar triangles is also included at this level (MoEYS, 2018d).

In upper secondary education, geometry becomes more abstract and analytical. It consists of two major concepts: (1) plane geometry focusing on algebra, analysis, and transformation, and (2) space geometry focusing on algebraic geometry and analysis, including the analysis of space geometry and conic sections. The

curriculum emphasises plane geometry through coordinate systems, where students analyze coordinates of points, linear and circular equations, and solve inequalities using graphical methods. They also explore vector geometry, performing operations with vectors and applying them to solve geometric problems. A significant part of the curriculum includes transformations in the plane, such as translation, reflection, rotation, and dilation (homothety), which help students understand geometric movements and symmetry (MoEYS, 2018f, 2018g).

In addition to plane geometry, upper secondary-level students study space geometry, which involves analyzing the relationships between lines and planes in 3D space. Concepts such as parallelism and orthogonality are also covered. Moreover, students work with vectors in space, including scalar products, and learn to formulate equations of planes, lines, and spheres. The study of conic sections (parabolas, ellipses, and hyperbolas) further develops their ability to connect algebraic expressions with geometric figures, completing their CU of geometry (MoEYS, 2018f, 2018g).

Even though the curriculum was structured, some content was still too advanced for students in general education. It is necessary to integrate ICT into teaching and learning to assist and support teachers and students in exploring more in-depth, as recommended by international curriculum experts and a report on the curriculum review of Science, Math and ICT under a project of the Upper Secondary Education Development Program (USESDP) in response to the needs of the global market (NIEI & PRI, 2021). While the curriculum was developed with specific ICT subjects and is focused merely on ICT for computer literacy, administrative support, communication, and presentation, the integration of ICT into mathematics content as a tool to support teaching and learning has not yet been included in the curriculum and textbooks or implemented in the actual teaching practices.

#### **1.1.4 Mathematics Assessment**

Cambodia's 2022 PISA results show that only 12% of students reached level 2 or above in mathematics, the baseline proficiency level or higher. The indicator of level 2 highlighted that students can recognize situations where they need to design simple strategies to solve problems, including running straightforward simulations involving one variable as part of their solution strategy. They can extract relevant information

from one or more sources that use slightly more complex modes of representation, such as two-way tables, charts, or two-dimensional representations of three-dimensional objects (MoEYS, 2024). Students can demonstrate a basic understanding of functional relationships and solve problems involving simple ratios at this level. They can interpret results literally compared to the PISA results of 2017; the proportion of students who scored at level 2 or higher in mathematics did not change significantly, from 9.9% in 2017 to 12% in 2022 (MoEYS, 2024). It is important to note that students achieve below the frequency level of about 88%, which indicates that Cambodian students are poor at least at this level of mathematics. The table below shows the percentage of students by proficiency level in mathematics in Cambodia’s PISA results in 2022.

*Table 3. Percentage of students by proficiency level in mathematics*

<b>Mathematics</b>		
<b>Cut Point Scores</b>	<b>Level</b>	<b>Percentage</b>
Below 233.17	Below 1c	7.60
From 233.17 to Below 295.47	1c	20.60
From 295.16 to Below 357.77	1b	33.70
From 357.77 to Below 420.07	1a	26.10
From 420.07 to Below 482.38	2	9.50
From 482.38 to Below 544.68	3	2.20
From 544.68 to Below 606.99	4	0.30
From 606.99 to Below 669.3	5	0.00
At or Above 669.3	6	0.00

Source: A Country Report PISA 2022 Results for Cambodia (MoEYS, 2024).

There are many possible approaches for enhancing students' learning achievement highlighted by MoEYS, such as the implementation of the key points of the prime minister<sup>3</sup>, implementation of school reform, teacher development through

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<sup>3</sup> The Prime Minister of Cambodia provides the key messages to MoEYS to take action to improve student learning outcomes and strengthen the quality of public education institutions from kindergarten to secondary schools through the implementation of four mechanisms: (1) strengthening school governance; (2) reviewing and revising the curriculum and extracurricular activities in accordance with the needs of the students to enhance their

pedagogical school reform, review of textbooks and improvement of teaching methods, increase community participation, promote school governance and support, curriculum development, promote school health, and provide student learning support. Among them, providing professional development for trainers and lectures, emphasizing the mastery of subject content, pedagogical skills, and technology have been addressed, meaning utilizing ICT in subject content for teacher development and student learning improvement should be considered (MoEYS, 2024, p. 45).

Besides PISA's results in 2022, the National Learning Assessment (NLA) report by MoEYS also highlighted that, on average, grade 8 students in the year 2021–2022 could correctly answer 42.5% of overall mathematics problems, with 46% of the students solving problems related to the geometry domain. The geometry contents covered five sub-contents: circle, rectangle, polygon area, lines and chords of a triangle, and triangle comparison (MoEYS, 2023), which were embedded in all geometry sub-contents in grade 8. The results show that the sub-content for circle performed higher than average (57.7%); the others performed below average: 49.3%, 33.3%, 32.4%, and 29.4% (the average is 50%) for rectangle, polygon area, lines and chords of triangles, and triangle comparison, respectively. The overall percentage of correct answers in the geometry domain by sub-content is shown in Figure 1.

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knowledge, discipline, morality, and behavior; (3) taking care of students' health through a nutrition programme for children and quality control of school food; (4) promoting and encouraging active involvement of parents, guardians, and the community in education in line with the slogan "State and Community Partnership for Education."

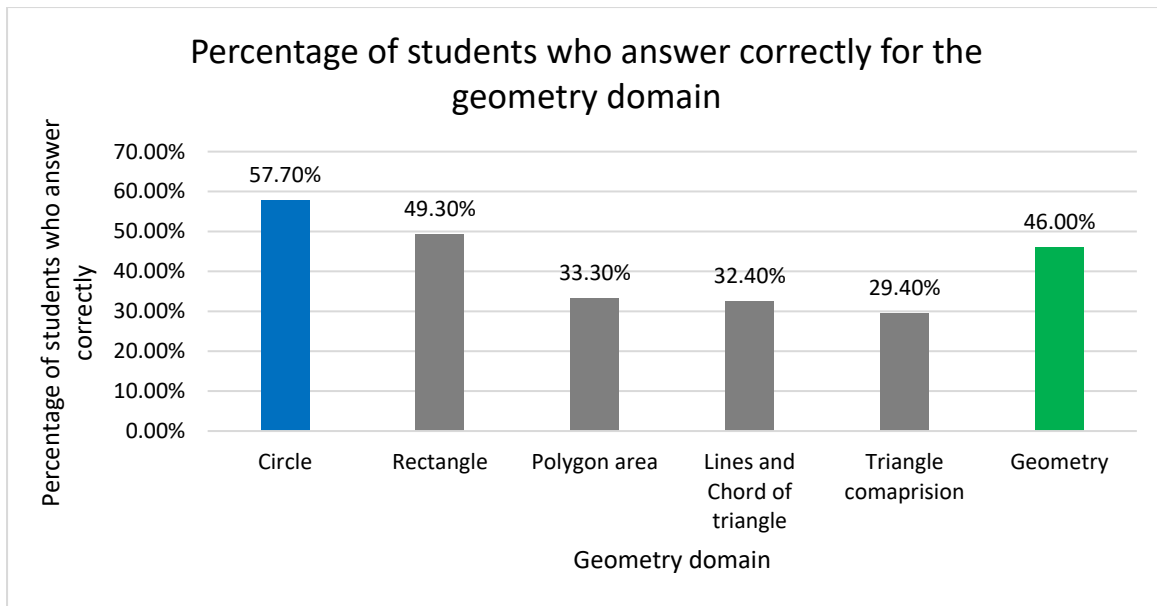


Figure 1. The overall percentage of correct answers for the National Learning Assessment in grade 8

Source: 2022 Grade 8 National Learning Assessment (MoEYS, 2023).

Another report on the national learning assessment in the year 2018, the result of grade 11 students' achievement revealed that a large proportion of students (72%) are below the basic proficiency level in mathematics; 18% are at the basic level, and the other 10% are at the proficient and advanced level. These findings require prompt action at the school level to create an efficient strategy to increase student achievement and reduce inequity in education (MoEYS, 2019b).

Furthermore, the analysis of the national exam report in the academic year 2021–2022 also revealed that, on average, 45.76% of students were able to answer mathematics problems across seven subject contents: limitation, combination, complex, integral, vector, differential equation, and function (MoEYS, 2022). Figure 2 shows the national examination results of grade 12 in the academic year 2021–2022.

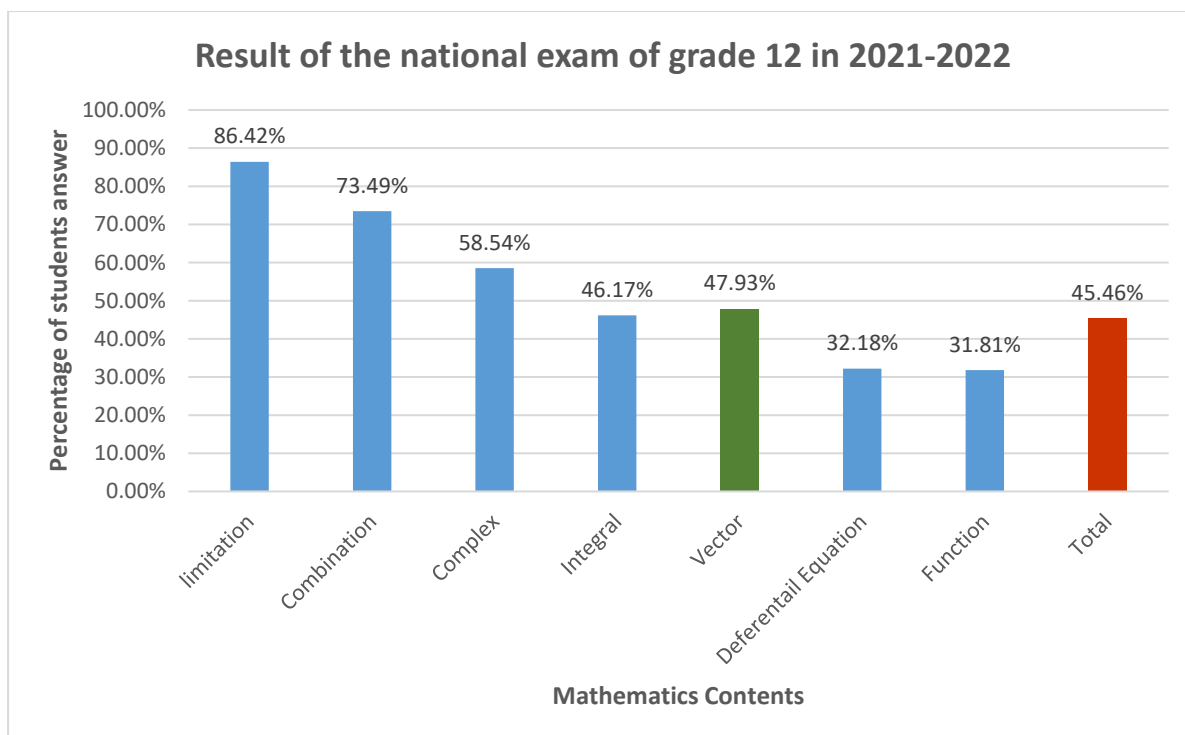


Figure 2. Result of the national examination of grade 12 in the academic year 2021–2022

Source: The author developed based on the national examination report of grade 12 in 2021–2022 (MoEYS, 2022).

Figure 2 shows that students who can solve the mathematics problems in the national exam are only 45.76% of students in the seven subject contents. Among them, only the vector content was selected to construct a test item included in the examination paper in the geometry domain. Even though the number of students who can answer correctly for the vector content (47.93%) is higher than that for the two other contents (differential equation and function), it is still below the overall average (MoEYS, 2022). These results indicate that students performed poorly in solving geometry problems.

Based on the above assessment results and the analysis report, it can be concluded that Cambodian students have poor performance in mathematics, including geometry, as highlighted in the PISA, NLA, and the national examinations. Moreover, geometry problems deal with shapes and figures, which contain some abstract information; students are required to have better visualization and representation skills to improve their ability to solve geometry problems (Kasik, 2019; Lowrie et al., 2019;

Orozco, 2017; Zhang et al., 2023). Although vectors scored higher than other topics (differential equations and functions) in the national exam, geometry remains a problematic area for Cambodian students, because it combines shape (spatial understanding) and algebra (symbolic reasoning). Additionally, the Cambodian mathematics curriculum, specifically geometry, lacks a clear progression from basic to advanced vector concepts, leading to weak CU. External assessments, such as NLA and PISA, confirm that students struggle with geometry, particularly in tasks that require visual representation and spatial reasoning. Therefore, geometry continues to be a significant learning challenge despite not having the lowest score. Moreover, the author also takes into consideration other related factors.

### 1.1.5 The Use of ICT in Mathematics Education

MoEYS formed a technical team to improve science and mathematics education in 2018. This team is led by the Department of Information and Technology (DIT) as part of the USESDP-2 project. They are responsible for exploring available ICT, such as open-source (Apps/dynamic software), that can be integrated into science and mathematics subjects to support teaching and learning in the actual classroom. As a result of their work, the team developed a teaching guidebook titled **“Guidebook on Using Tablets to Support Science and Mathematics Teaching”** (in the Khmer language) in 2020 (MoEYS, 2020a). The guidebook is a comprehensive resource for science and mathematics subjects. It covers five subjects: Mathematics, Physics, Chemistry, Biology, and Earth Science. The teams identified and allocated the available dynamic software packages to each relevant subject, as shown in Table 4.

*Table 4. The available dynamic software packages are suitable for integration into science and mathematics*

No	Subject	Available software
1	Mathematics	1. Simple Sequence Calculator 2. Desmos 3. 3D Calculator 4. Math Solver 5. GeoGebra 6. Graphing Calc

		<ul style="list-style-type: none"> <li>7. Photomath</li> <li>8. RealCal</li> <li>9. Sci Calc</li> </ul>
2	Physics	<ul style="list-style-type: none"> <li>1. Electronics Toolkit</li> <li>2. PhET Interactive Simulations</li> <li>3. Resources-Drawing</li> <li>4. Khan Academy</li> </ul>
3	Chemistry	<ul style="list-style-type: none"> <li>1. Chemistry Advisor</li> <li>2. Chemistry Practicals</li> <li>3. IUPAC Nomenclature</li> <li>4. KingDraw Chemical Structure</li> </ul>
4	Biology	<ul style="list-style-type: none"> <li>1. 3D biology</li> <li>2. Animal 4D</li> <li>3. Bacteria Interactive Educational VR</li> <li>4. Biology Grade 12</li> <li>5. Biology Mitosis &amp; Meiosis</li> <li>6. Cell Word</li> <li>7. Circulatory System</li> <li>8. Eye Anatomy 3D</li> <li>9. Human Anatomy</li> <li>10. Khmer Academy</li> </ul>
5	Earth Science	<ul style="list-style-type: none"> <li>1. Star Chart</li> <li>2. Star Walk</li> <li>3. Google Earth</li> <li>4. Solar Walk Lite</li> </ul>

Source: Guidebook on Using Tablets to Support Science and Mathematics Teaching (MoEYS, 2020a).

In mathematics, the technical teams have found nine possible software packages suitable for use in mathematics content. The team developed the teaching guide and provided cascade training to selected secondary resource schools (SRS) nationwide. Through the teaching process and observation, mathematics teachers

(trainees) frequently mentioned GeoGebra software as a potential software to use in geometry teaching to overcome students' challenges in solving mathematical problems, especially in the geometry domain, and to enhance students' academic performance. Furthermore, scholars mentioned that GeoGebra software is a suitable dynamic geometry software to integrate into teaching and learning mathematics to overcome students' difficulty in solving geometry problems (Gurmu et al., 2024; Olivares, 2024; Samura, 2023; Sieng & Thai, 2024a; Siswanto et al., 2024). However, the technical department or other researchers have not yet measured and researched the effectiveness of integrating GeoGebra software into teaching and learning geometry in the actual classroom in the Cambodian context after the training.

## **1.2 PROBLEM STATEMENT**

Cambodian students have poor abilities in solving geometry problems, as shown by various documents and reports. First, the results of PISA in 2022 show that only 12% of students achieved level 2 or higher in mathematics, while the average for ASEAN and the OECD is 42.6% and 68.9%, respectively. The students' low achievement scores are also observed in mathematics subjects, specifically in the geometry domain (MoEYS, 2024). Second, the NLA conducted in November 2021 revealed that students' achievement in mathematics remained low compared to the NLA of 2016 (UNICEF, 2022), in which the overall score was only 38% (Bhatta et al., 2022). The geometry domain had the lowest scores compared to the other domains (algebra, statistics, measurement, and numbers), with only 35% of grade 6, 46% of grade 8, and 38% of grade 11 students completing the assessment test correctly (Bhatta et al., 2022; MoEYS, 2019b). Thirdly, the grade 12 national examination analysis for the academic year 2020–2021 found that only 47.93% of students could solve geometry problems correctly, particularly vector content (MoEYS, 2022). All these indicators reveal insufficient student achievement in geometry.

Factors that influence students' low achievement scores include educational policy, the curriculum, school-related issues, personal factors, teachers' knowledge of geometry, the development of effective teaching and learning materials, teaching instruction, and social factors (Hasibuan et al., 2019; Kholid, 2022; Purevdorj, 2016; Teodorović, 2011; Ulandari et al., 2019; Wang & Kao, 2022). Among the factors is integrating dynamic geometry software (DGS) into teaching and learning geometry content, particularly GeoGebra software, which is a possible approach to help students

overcome their difficulties in solving geometry problems by improving visualization skills (Gurmu et al., 2024; Hohenwarter et al., 2010; Hohenwarter & Fuchs, 2004). In 2020, MoEYS developed guidebooks and cascade training, including GeoGebra software, for schoolteachers to integrate into mathematics content, especially geometry (MoEYS, 2020b). Up to now, mathematics teachers at SRSs have received cascade training on using tablets to support science and mathematics teaching. Even though trainees received nine software packages for mathematics, including GeoGebra software, they received training material for further professional development on using those software. Teachers' understanding of utilizing GeoGebra software is still limited in terms of both methodology and content knowledge. Moreover, there is a lack of research that measures the effectiveness of using GeoGebra in teaching and learning mathematics, particularly geometry. Therefore, this study aims to investigate the integration of GeoGebra as an intervention in geometry lessons.

Another important factor is that the Cambodian mathematics curriculum itself might not be well organized and coherent. To maximize the effect of the experimental lessons using DGS, we had to rectify some of the curriculum's weaknesses when developing the intervention using DGS. Therefore, it is important to analyze the curriculum. It should consistently align with broader educational goals, objectives, contents, sub-contents, and learning outcomes. In other words, curriculum coherence is the most dominant predictor for student achievement because it refers to the connection and logical progression of content, skills, and learning outcomes within a curriculum framework, syllabus, textbook, and assessment (Cuoco & McCallum, 2018; Prawat & Schmidt, 2006; Schmidt et al., 2002). Thus, this study aims to investigate the integration of DGS into mathematics teaching from the perspective of curriculum coherence.

### **1.3 RESEARCH OBJECTIVES AND QUESTIONS**

As indicated in the background and problem statement above, the main purpose of this study is to measure the effectiveness of integrating GeoGebra software in teaching and learning geometry at upper secondary schools. Four main objectives are addressed as follows:

1. To identify the students' difficulties in learning geometry.

2. To identify the most suitable dynamic geometry software approach to overcome the students' learning difficulties in geometry.
3. To analyze the Cambodian mathematics curriculum, focusing on geometry in upper secondary schools, and to find out which topic is critical.
4. To measure the effectiveness of coherent geometry lessons integrating GeoGebra software at upper secondary schools in Cambodia.

In response to the research objectives, four research questions have been addressed as follows:

1. What are the students' difficulties in learning geometry?
2. Which approaches does dynamic geometry software use to address the above learning difficulties?
3. What is the current geometry curriculum in upper secondary schools in Cambodia, and which topic is critical?
4. To what extent does a developed coherent mathematics lesson integrating GeoGebra software improve students' geometry learning in upper secondary schools in Cambodia?

#### **1.4 SIGNIFICANCE AND LIMITATIONS OF THE STUDY**

The results of this study provide the following benefits:

- They are relevant not only for the Cambodian context but also for other countries worldwide where students face the same difficulties in solving geometry problems.
- They can serve as a valuable guide for curriculum developers and textbook writers to consider for future revision.
- They can support teachers in preparing teaching and learning materials and promote students' active involvement in mathematics teaching and learning.

Nonetheless, this study faces some limitations. The following points may affect the generalizability of findings:

- **Number of schools:** The study was conducted with a limited sample of grade 10 students, and two upper secondary schools in Cambodia were selected. The findings might not be generalizable to all the country's upper secondary schools or different educational contexts.

- **Intervention period:** The intervention period for the integration of GeoGebra software into the teaching and learning process was short (five weeks). An extended period may be needed to fully investigate the impact of GeoGebra software on students' geometry learning outcomes.
- **Mathematics topics:** Implementing GeoGebra software with other content in the geometry domain would provide more insights into how it could improve mathematics education as a whole.

## 1.5 STRUCTURE OF THE DISSERTATION

This dissertation consists of the following six logically connected chapters:

**Chapter 1. Introduction:** Presents an introduction and rationale focusing on the education system in Cambodia. It provides an overview of education in Cambodia, the curriculum frameworks of general and technical education, mathematics education, mathematics assessment, and the use of ICT in mathematics education, especially the integration of GeoGebra software into mathematics in the geometry domain. It also outlines the problem statements, objectives, questions, significance, and limitations of the study and the dissertation's structure.

**Chapter 2. Literature review:** Presents the global discussion on students' difficulties in learning geometry through visualization and its components. It also discusses the implementation of visualization at different educational levels and the relationship between its components: visual perception, visual-spatial skills, and visual representation. This chapter also discusses the approaches to integrating dynamic geometry software to overcome students' learning difficulties in geometry. Additionally, CU is addressed, as it is closely interrelated with visualization. At the end of this chapter, a summary and conceptual framework of the study are provided.

**Chapter 3. Methodology:** Explains the two methods of the study: (1) curriculum analysis, describing the data source and data analysis with a focus on coherent curriculum, and (2) field survey, describing the procedure of implementing the intervention of experimental lessons to overcome students' difficulties in learning geometry by integrating GeoGebra software. This methodology also covers the research location and participation, research design and procedure, instrument design, data collection, data analysis, and ethical considerations.

**Chapter 4. Result:** Presents the results of the curriculum analysis and field survey, such as the improvement in students' achievement through intervention lessons and the semi-structured interviews by curriculum developers, teachers, and students.

**Chapter 5. Discussion:** Presents the discussion and compares the results of this study with those of previous research.

**Chapter 6** summarizes the main findings for each research question and provides recommendations for future research.

The figure below shows the construction of the thesis:

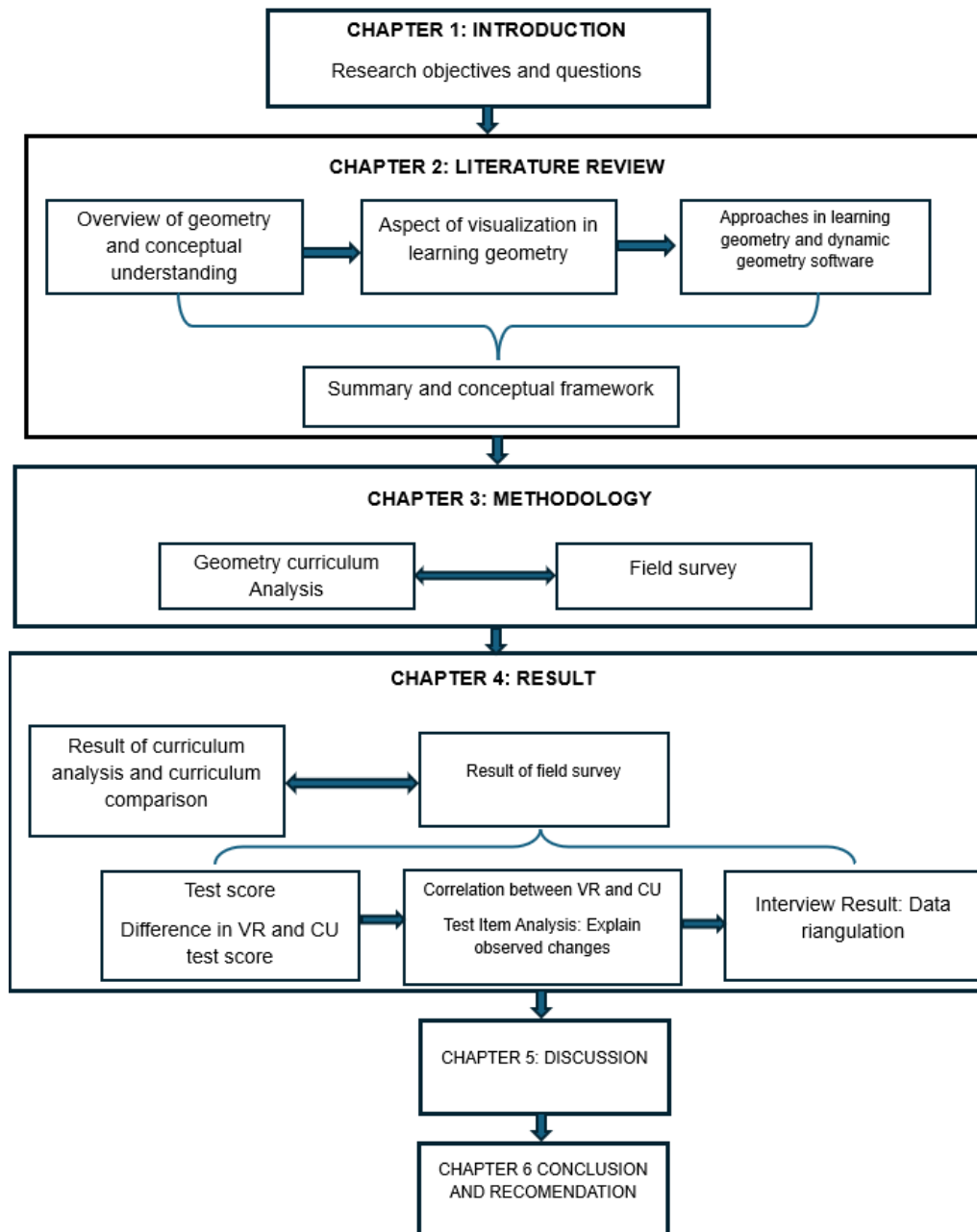


Figure 3. Construction of thesis

## **CHAPTER 2: LITERATURE REVIEW**

The literature review focuses on geometry learning, visualization, and approaches to enhance visualization, aiming to achieve the study's objective. The end of this chapter provides a summary and conceptual framework of the research process.

### **2.1 OVERVIEW OF GEOMETRY**

#### **2.1.1 Geometry**

Geometry is a domain of mathematics that studies shapes and figures, including their properties and relationships (Biber et al., 2013). The word “geometry” comes from the Greek words “geo,” meaning “earth,” and “metron,” meaning “measurement.” It also plays a pivotal role in human activities, influencing technology, architecture, art, engineering, and physics (Kerlow, 2009; Silmi Juman et al., 2022; Singh & Kumar, 2022; Tang & Ho, 2020). For example, it is applied in many areas like structure design, computer graphics development, robotics, video game development, and space navigation. Geometry is divided into two main branches at the school level for general education: (1) Plane geometry, which is concerned with shapes like circles, triangles, and polygons that exist in two-dimensional space, and (2) Space geometry, which deals with three-dimensional objects like cubes, spheres, and cylinders (Boyer & Merzbach, 2011).

The study of geometry equips students with essential skills, including problem-solving, reasoning, logical thinking, and critical analysis. These skills are valuable in daily life and across various scientific disciplines. In daily life, geometry helps individuals interpret spatial information, make informed decisions, and approach real-world challenges methodically (Dimla, 2018; Van de Walle et al., 2022; Walle et al., 2010). For example, when navigating directions, designing spaces, or estimating measurements. Furthermore, learning geometry emphasizes exploring different representations, such as virtual manipulatives, written math formulas, and verbal explanations, which help students build mathematical concepts and develop critical thinking (Silmi Juman et al., 2022). Students with advanced knowledge in geometry are vital, as they form the foundation for other mathematical fields, such as algebra,

calculus, trigonometry, and statistics. Early exposure to geometry enhances spatial awareness and dynamic visualization skills, which are essential for grasping more abstract mathematical concepts and are more critical for success in higher-level mathematics (Eshetu et al., 2023). Additionally, a solid understanding of geometry helps address practical problems in daily life, such as measuring distances, creating drawings, and interpreting figures or maps (Hershkowitz et al., 1998). By solving geometric problems, students learn to approach complex issues methodically and think critically (Roath, 2015).

In Cambodia, geometry is introduced in general education from early to upper secondary school. Geometry practice in Cambodia aims to equip students with essential mathematical problem-solving skills (MoEYS, 2016). MoEYS has developed more advanced geometry at the upper secondary level, focusing on Euclidean and coordinate geometry. At this level, students engage with complex problem-solving tasks, proofs of theorems and definitions, and applications of geometric shapes and algebra, which are more related to daily life and real-world contexts. Moreover, students gain new complex knowledge of plane and space geometry, which continues their professional skill development to the university level; this impacts their university choice and career development (MoEYS, 2018h, 2018i).

It is important to note that the Cambodian mathematics curriculum at upper secondary school is divided into two tracks: social science and science. The contents of the social science track are allocated less than those of the science track. Some content of the science track is expanded from the social science track by adding more sub-contents and increasing the teaching hours. However, no clear official guidelines or regulations guide teachers between the two tracks. Most teachers have applied their teaching methods based on their experiences, specifically focusing on the national examination (Ngel, 2023). Some content is skipped by teachers, making students lose connection with the related content and conceptual knowledge because critical content and knowledge are missed. Table 5 presents the contents allocated to upper secondary school education.

Table 5. Geometry curriculum at upper secondary schools in Cambodia

No.	Contents	Sub-contents			
		Science track	Hours	Social science track	Hours
<b>Grade 10</b>					
1	Geometry in a plane	<ol style="list-style-type: none"> <li>1. Coordinate points in a plane</li> <li>2. Linear equations</li> <li>3. Circular equations</li> <li>4. Solving the problem of inequality using graphs</li> </ol>	22	<ol style="list-style-type: none"> <li>1. Coordinate points in a plane</li> <li>2. Linear equations</li> <li>3. Circular equations</li> </ol>	18
2	Vector in a plane	<ol style="list-style-type: none"> <li>1. Vectors and operations on vectors</li> <li>2. The practice of a vector</li> </ol>	24	<ol style="list-style-type: none"> <li>1. Vectors and operations on vectors</li> <li>2. The practice of a vector</li> </ol>	25
3	Transformation in a plane	<ol style="list-style-type: none"> <li>1. Moving transformation</li> <li>2. Symmetric transformation</li> <li>3. Rotate transformation</li> <li>4. Homothety transformation</li> </ol>	24	The contents and sub-contents were not addressed in the curriculum.	
<b>Grade 11</b>					
1	Geometry in space	<ol style="list-style-type: none"> <li>1. The shape of geometry in space</li> </ol>	20	The contents and sub-contents were not	

		<ol style="list-style-type: none"> <li>2. Line and parallel planes in space</li> <li>3. Orthogonal in space</li> </ol>		addressed in the curriculum.	
<b>Grade 12</b>					
1	Conic	<ol style="list-style-type: none"> <li>1. Parabola</li> <li>2. Ellipse</li> <li>3. Hyperbola</li> </ol>	18	The contents and sub-contents were not addressed in the curriculum.	
2	Analyzing geometry in space	<ol style="list-style-type: none"> <li>1. Vectors in space</li> <li>2. The scalar product of two vectors</li> <li>3. The scalar product of two vectors in space</li> <li>4. Plane equations</li> <li>5. Line equations in space</li> <li>6. Sphere equations</li> </ol>	28	The contents and sub-contents were not addressed in the curriculum.	

Sources: Detailed mathematics curriculum for general and technical education (MoEYS, 2018h, 2018i).

The geometry content of the science track is more than that of the social science track in upper secondary education. They are complex for students in general education to learn. More content requires students to have strong visualization skills to understand the concepts. There is a need for greater integration of interactive

teaching methods and ICT to enhance students' understanding and engagement, particularly at the upper secondary level, and to overcome students' difficulties in learning geometry at this educational level (MoEYS, 2016; Sieng & Thai, 2024b).

The teaching hours allocated to the geometry domain at each educational level appear to be inappropriate compared to other mathematical domains. The geometry teaching hours in detailed mathematics for general and technical education were allotted, as shown in Table 6.

*Table 6. The percentage of teaching hours allocated for the geometry domain in Cambodian mathematics compared to other domains for the science track.*

<b>Grade</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
No. of domains	5	5	5	6	6	6	6	6	6	8	8	8
Total hours	245	245	245	210	210	210	245	245	245	210	210	210
Geometry	32	34	25	57	70	104	112	123	128	70	20	46
%	13	14	10	23	29	43	46	50	52	29	8	19

Sources: The author developed the table based on a detailed Cambodian mathematics curriculum.

The geometry domain is allotted more teaching hours than other domains in lower secondary education, with 112 hours in grade 7, 123 hours in grade 8, and 128 hours in grade 9. In primary education, geometry had fewer teaching hours than other domains, from grades 1 to 3. Moreover, upper secondary education was given the least amount of teaching hours for the geometry domain in grade 11, more specifically, in the science track (MoEYS, 2018f).

Geometry content was not allocated in the social science track for grades 11 and 12. It is important to note that the science and social science tracks were classified separately in upper secondary education at grades 11 and 12. Table 7 shows the geometry teaching hours in Cambodia's general and technical education for the social science track.

*Table 7. The percentage of teaching hours allocated for the geometry domain in Cambodian mathematics compared to other domains for the social science track*

<b>Grade</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
No. of Domains	5	5	5	6	6	6	6	6	6	8	8	8
Total hours	245	245	245	210	210	210	245	245	245	175	175	175
Geometry	32	34	25	57	70	104	112	123	128	43	0	0
%	13	14	10	23	29	43	46	50	52	18	0	0

Source. The author developed the table based on a detailed mathematics curriculum.

Even though the geometry domain was allocated more teaching hours than other domains in a detailed mathematics curriculum and organized by adding up knowledge and skills, the students' achievement was still poor in solving geometry problems, as shown in the national examination and NLA reports (Bhatta et al., 2022; MoEYS, 2022, 2023). Therefore, the Cambodian mathematics curriculum, particularly the geometry domain, was also analyzed.

### **2.1.2 Aspect of Curriculum Coherence**

Oates (2017) suggests that “curriculum coherence” is crucial and linked with high-performing systems. Curriculum coherence refers to more than just the simple, common use of the word “coherence”; a curriculum is considered “coherent” when the assessment, pedagogy, teaching content, textbooks, and national curriculum contents align with and reinforce one another (Schmidt & Prawat, 2006a). Coherence means that all these aspects are interrelated.

However, different researchers have defined curriculum coherence in different ways. For instance, Newmann et al. (2001) defined curriculum coherence as the linkage and interconnectedness of the various components of a curriculum, such as the learning objectives, instructional materials, teaching methods, and assessments. These should be organized to ensure that all components work together logically and are integrated to support student learning and achievement. In summary, curriculum coherence provides students with a clear and consistent educational experience by avoiding gaps, repetitions, or contradictions.

Similarly, Schmidt et al. (2002) defined curriculum coherence as the logical and sequential organization of the content standards and curricula across a period. It involves articulating a sequence of topics and performances that reflect the discipline's hierarchical nature. Coherent content standards and curricula are designed to progress from simple content to more profound and abstract content, which is inherent in the discipline.

Bateman et al. (2007) defined curriculum coherence as the degree to which the intended learning outcomes (instructional objectives), the instructional processes (learning activities), and the assessments (formative assessments and summative evaluations of the student learning) are connected or aligned. They are intricately related and connected, with a focus on objectives, activities, and learning outcomes.

Wang and McDougall (2018) defined curriculum coherence as the degree to which a curriculum is organized logically and meaningfully. Here, curriculum coherence builds on the student's prior knowledge and provides a clear learning progression, with each content or concept building on what the student has learned. This coherence helps students develop an understanding of one topic in connection with another. It also allows teachers to plan and deliver instruction more effectively by providing a clear framework for what should be taught and when a given topic could be introduced.

Reeves and McAuliffe (2012) defined curriculum coherence as the degree to which domain-specific or disciplinary content is systematically presented to learners in terms of the conceptual coherence of its organization. In other words, it is the extent to which the curriculum is structured in a way consistent with the logical and hierarchical nature of the disciplinary content from which it is derived. This coherence includes the ordering relationships between content elements, the rationale for that order, and the connections and interlinkages between different contents and sub-contents within the subject field.

Confrey et al. (2017) defined a coherent curriculum as a well-structured and interconnected curriculum that ensures students have a clear and logical progression of learning experiences. This coherence can be achieved by aligning content across grades, coordinating content within a grade, and ensuring consistency in sequencing concepts and activities.

Oates (2017) defined curriculum coherence as the coordination and consistency of national curriculum content, teaching materials, pedagogical

approaches, assessment methods, and incentives within the education system. This coherence means linking and integrating various elements within an education system.

These discussions of curriculum coherence can be summarized under three points. First, curriculum coherence means the organization of content, such as grades or grade bands, content domains, and cognitive domains across stages (Reeves & McAuliffe, 2012; Schmidt et al., 2002). This coherence can be called the structural aspect of curriculum coherence.

Second, it entails connections between different contents within a specific topic (Reeves & McAuliffe, 2012; Wang & McDougall, 2018). This aspect of coherence means that the contents are sequenced coherently and logically, and such a sequence of contents supports learning progression across grades. In other words, students build their knowledge and skills from one grade to the next (Confrey et al., 2017; Schmidt & Prawat, 2006b). This coherence can be called the content-relation aspect of curriculum coherence.

Third, it involves the connection between objectives, instructional processes, learning outcomes, and assessment (Bateman et al., 2007; Newmann et al., 2001). The curriculum's goals and objectives outline what will be achieved through the educational process. This relationship highlights the importance of aligning the goals of the curriculum with its actual implementation and leads to the intended outcomes. This relationship contributes to effective student learning experiences (Oates, 2017). This can be called the pedagogical aspect of curriculum coherence.

The above three points are related to internal coherence. Through a curriculum with internal coherence, students are more likely to experience a logically connected progression of learning experiences, which can contribute to a deeper understanding and mastery of the content (Wang & McDougall, 2018). Moreover, the curriculum can be aligned with social needs and expectations. This is called external coherence because social needs and expectations do not belong to the education itself. Therefore, both internal and external coherence form the overall coherence of the curriculum. However, this research will focus only on internal curriculum coherence because it is assumed to be more directly concerned with student learning achievement. Table 8 shows all the aspects of curriculum coherence.

Table 8. Aspects of curriculum coherence

Type	Aspects	Descriptions
Internal coherence	Structural	Grades or grade bands, content domains, and cognitive domains
	Content relation	Different contents within a specific topic
	Pedagogical	Objectives, instructional processes, learning outcomes, and assessment
External coherence		Relationship with social needs and expectations

Source. The author adopted the information from Sieng and Baba (2024).

Table 8 shows three interrelated aspects: structural, content relation, and pedagogical. These relationships among the three aspects mean that an effective curriculum design considers the interplay between these three aspects to create a cohesive and meaningful learning experience for students. The content relation aspect will be the focus of this study because it can most effectively direct student learning achievement by ensuring that the curriculum is organized coherently and meaningfully (Confrey et al., 2017; Watson & Ohtani, 2015). By establishing meaningful connections between concepts, educators can help students see the connectedness and significance of their learning, leading to a deeper understanding and retention of the material.

In this study, the structural and content relation coherences are focused. Thus, curriculum coherence is defined as the relationship between corresponding domains, sub-domains, contents, sub-contents, and learning outcomes within and across grades; they are all arranged in order as a whole.

### 2.1.3 Conceptual Understanding

Conceptual understanding (CU) goes beyond rote memorization of formulas or calculation procedures. It involves comprehending principles, relationships, and reasoning processes. For teachers, a deep CU is crucial for effective instruction. Howe (1999) studied the knowing and teaching of elementary mathematics and emphasized that teachers with a profound knowledge of fundamental mathematics are better equipped to teach concepts in depth, make connections across topics, and anticipate student difficulties.

In geometry teaching, teachers' CU is vital for developing students' reasoning and spatial thinking. Effective geometry teaching requires teachers to justify geometric relationships and encourage logical reasoning (Stylianides & Ball, 2008). Many teachers rely on procedural instruction rather than helping students understand the underlying principles of geometric rules. Thus, teachers must develop reasoning skills to guide students in constructing valid arguments and justifications (Fujita & Jones, 2007). Irshid et al. (2023) studied the effect of CU principles-based training programs on the enhancement of mathematics teachers' pedagogical knowledge and defined CU as the ability to connect mathematical ideas and apply them in various contexts. Their study highlighted that CU could significantly enhance teachers' pedagogical knowledge and help them improve explanations, use multiple representations, and understand students' thinking.

Clements (1992) highlighted that teachers need to deeply understand geometric concepts such as shape classification, symmetry, congruence, and transformation. Strong content knowledge allows teachers to make meaningful connections across concepts and respond to students' misconceptions. Without this, instruction often becomes fragmented and overly focused on memorization.

Arcavi (2003) and Presmeg (2020) mentioned that teachers need the ability to use different forms of visualization during instruction because geometry relies heavily on visual thinking. They should be skilled in using a variety of representations, including diagrams, mental imagery, and digital tools. Visualization is not just for demonstration; it plays a central role in reasoning, solving geometry problems, and allowing real-time exploration of geometric relationships. Therefore, a strong CU of geometry enables teachers to foster student reasoning, explore interconnected geometric properties in reasoning, relational learning, and visualization, which is crucial for improving geometry education.

For students, CU refers to a learner's comprehensive grasp of the concept, where knowledge is retained and deeply internalized, interconnected, and transferable across various contexts. For instance, Leinwand et al. (2014) mentioned that students exhibit CU when they can represent mathematical situations differently, recognize connections among concepts, and explain why the procedure works. In other words, it is a foundation for developing mathematical reasoning and solving problems. CU

must proceed and coincide with instruction on the method, emphasizing the need for students to connect concepts and the procedure of solving problems.

Yurekli et al. (2020) emphasized that CU involves students recognizing when and why to apply specific mathematical concepts. It goes beyond mere procedural knowledge, focusing instead on the ability to connect and apply ideas meaningfully. For example, given a triangle on a coordinate plane, the student might draw a perpendicular from a vertex to the opposite side to determine the height, using coordinate geometry or even the distance formula if needed. This concept shows flexibility and a deep knowledge of geometric principles.

Rupnow and Fukawa-Connelly (2024) defined CU as students' ability to recognize when to use specific concepts in solving problems, explain their applicability, and relate multiple representations of a concept. This CU involves students' proficiency in identifying appropriate mathematical concepts during problem-solving, clearly articulating why these concepts are relevant, and effectively connecting different representations, such as visual, symbolic, and verbal forms, to deepen comprehension and facilitate meaningful learning.

Gurmu et al. (2024) addressed that CU is essential for fostering logical reasoning, problem-solving abilities, and the practical application of mathematical knowledge. It requires implementing appropriate methods that actively engage students and support them in becoming autonomous, critical thinkers and self-directed learners.

These studies mainly focus on three points. First, the teachers and students are able to justify, select, and provide the reason through the mathematical procedure. These refer to the abilities of the teachers and students to understand mathematical procedures and reasoning (Clements et al., 1992; Fujita & Jones, 2007; Gurmu et al., 2024; Howe, 1999; Irshid et al., 2023; Leinwand et al., 2014; Rupnow & Fukawa-Connelly, 2024). Second, the research focuses on the conceptual development of geometric ideas, including how teachers and students construct and apply geometric properties flexibly and meaningfully (Arcavi, 2003; Gurmu et al., 2024; Presmeg, 2020; Yurekli et al., 2020). Third, it focuses on teachers and students' ability to connect with different forms of visualization, such as connecting procedures with visual, symbolic, and verbal representation (Arcavi, 2003; Irshid et al., 2023; Presmeg, 2020; Rupnow

& Fukawa-Connelly, 2024). In summary, CU can be condensed into three main points: the ability of students (1) to provide reasons, (2) to consider geometric ideas, and (3) to use various forms of visualization. To meet the research purpose of this study, **CU is defined as the students' ability: (1) to explain the mathematical concept following the mathematical procedure, including calculation (Properties); (2) to provide the justifications for the mathematical procedure (Reason); and (3) to use the mathematical expression to give the reason in relation to the visual representation (Representation).**

## **2.2 ASPECTS OF VISUALIZATION IN LEARNING GEOMETRY**

### **2.2.1 Visualization**

Manovich (2010) defined visualization as the mental ability to create, manipulate, and interpret images or representations of geometric shapes, figures, and their spatial relationships, either mentally or through diagrams. This ability is a crucial cognitive skill in mathematics, especially in geometry. It allows students to grasp geometric concepts, develop logical reasoning skills, and approach complex problems effectively. The complexity of geometric problems often lies in the need to visualize two-dimensional and three-dimensional shapes and their geometrical properties, relationships, and theorems (Baiduri et al., 2020). This cognitive demand can lead to difficulties for students when solving geometric problems.

Kumar (2019) defined the process and product of creation, interpretation, use of reflection upon pictures, images, and diagrams in our minds, on paper or with technological tools, to depict and communicate information, thinking about and developing previously unknown ideas, and advancing understandings.

Clements et al. (1992) and Alghadari et al. (2020) observed that students' poor visualization skills contribute to an insufficient ability to solve geometry problems. This, in turn, affects several educational outcomes, such as lower learning performance, lower achievement scores, and limited conceptual knowledge of geometry.

Scholars have emphasized that visualization in geometry is a multifaceted skill involving recognizing and interpreting geometric shapes and their features, which is essential for understanding spatial relationships (Baiduri et al., 2020; Manovich, 2010). Difficulties in this area can hinder students' ability to see patterns and details in geometry (Clements, 1992; Gal & Linchevski, 2010). This visualization can be called

**visual perception**, whereby students interpret and make sense of visual information received from their eyes.

Students' abilities are lacking in visualizing the mental manipulation of shapes, such as imagining rotations or transformations in two and three-dimensional shapes. This ability is crucial for solving complex geometric problems. This skill is essential for interpreting transformations and spatial reasoning tasks (Alghadari et al., 2020; Baiduri et al., 2020; Clements, 1992; Makamure & Jojo, 2021; Manovich, 2010). This visualization can be called **visual-spatial skills**, through which students perceive, analyze, and understand visual information in the environment and the spatial relationships between geometric shapes, theorems, and verbal expression, and create models.

Furthermore, students' abilities are lacking in externalizing thought through creating and using diagrams or digital tools to depict and communicate geometric ideas. It involves the visualization process and product, helping students develop and share their understanding (Alghadari et al., 2020; Clements, 1992; Kumar, 2019; Žakelj & Klancar, 2022; Zhang et al., 2014). This visualization can be called a **visual representation (VR)**, whereby students are able to recognize, perform mental operations, and externalize the representation.

Based on the literature review, it can be concluded that students' difficulties in solving geometry problems are due to they are lack of ability in three main aspects: visual perception, visual-spatial, and visual representation, which leads to low achievement, indicating a lack of **visualization skills**. All three are interrelated. This visualization in geometry can be defined as ***the ability to mentally and physically create, manipulate, and interpret geometric shapes and spatial relationships using mental imagery, diagrams, or digital tools to support understanding and solving problems.***

Next, we examine the three aspects of visualization.

### **2.2.1.1 Visual Perception**

Visual perception in geometry refers to the cognitive ability that allows students to interpret, analyze, and understand spatial relationships, shapes, and figures based on what they see (Bruce et al., 2014). This process is more than just receiving visual input; it involves the brain's active organization and interpretation of visual information to construct a coherent understanding of geometric figures (Cornsweet, 2012).

Several researchers have documented difficulties students face when these perceptual processes are underdeveloped. For example, Gal and Linchevski (2010) found that many students struggle to distinguish geometric objects when shapes overlap or cannot see and interpret different objects, leading to complications in the thinking process. These perceptual difficulties can obstruct students' ability to reason logically or solve geometric problems.

Sari and Slamet (2018) further explained that such challenges in perception can disrupt students' thinking processes, making it difficult for them to interpret or engage with mathematical content effectively. When students cannot perceive or organize visual information accurately, their ability to form mental representations of geometric concepts is hindered. This disruption affects their immediate understanding of a geometric problem and their ability to apply logical reasoning, recognize patterns, or connect visual inputs to abstract mathematical ideas.

Snowden et al. (2012) highlighted that visual perception in geometry involves several sub-skills, such as figure-ground perception (identifying a shape from its background), perception of spatial relationships (understanding how shapes relate in space), visual discrimination (noticing subtle differences between shapes), and visual memory (recalling and applying visual information). The difficulty in these areas can hinder students' ability to identify and work with geometric figures correctly.

Students face visual perception challenges, making it hard to distinguish between geometric shapes when they overlap or are presented in different ways or configurations (Gal & Linchevski, 2010). For instance, when students are shown a diagram where several triangles overlap, some students may struggle to identify the individual triangles due to figure-ground perception issues. This visual perception difficulty prevents them from effectively distinguishing shapes and interferes with their ability to engage in higher-level mathematical reasoning or problem-solving skills. All these problems mostly happen in primary education (Baluti et al., 2012; Gal & Linchevski, 2010; Richmond & Holland, 2010).

### **2.2.1.2 Visual-Spatial Skills**

Visual-spatial skills play a critical role in students' understanding of geometry, particularly in tasks that involve perceiving, interpreting, and mentally manipulating geometric figures and spatial relationships. Clements (1992) defined visual-spatial skills as the ability to perceive and mentally transform visual and spatial information.

These skills are essential for constructing and visualizing geometric shapes, understanding spatial relationships, and solving geometrical problems that require imagining shapes in different orientations. Many students, especially at the lower secondary level, face significant difficulties with visual-spatial tasks. For instance, students often struggle to mentally transform from 2D to 3D objects or to visualize how a 3D object would appear when unfolded into a flat shape.

Van Garderen (2006) argued that visual-spatial skills are fundamental for geometric constructions and crucial for reasoning about spatial transformations such as rotations, reflections, and translations. Students with strong visual-spatial skills are more likely to develop deep CU, while those with weaker abilities tend to rely on memorized procedures without grasping the underlying geometric principles.

Zhang et al. (2014) emphasized that students need to mentally track or imagine how a shape transforms, and this ability is essential for solving geometry problems and developing deep understanding. Their study reinforces the idea that visual-spatial reasoning is a key part of successful geometry learning. In contrast, students with limited visual-spatial skills struggled with these tasks, leading to poor performance in geometry.

Several studies have highlighted specific areas where students commonly encounter visual-spatial difficulties. Wahab et al. (2014) found that many students struggle to mentally visualize the hidden parts of 3D objects, such as the back, bottom, or interior of a cube when those parts are not visible from the given viewpoint. For example, if a cube is shown from the front, students might have difficulty imagining what the back side looks like or how all the sides connect together. Kalun et al. (2020) further found that many students have difficulty mentally rotating shapes or visualizing orientation changes, particularly when tasks involve multiple or complex transformations. This difficulty indicates a weakness in spatial visualization, a key component of visual-spatial skills. When students cannot mentally complete or fully picture the object, they cannot fully understand its structure, orientation, or geometric properties. This limitation prevents students from accurately solving problems that require them to reason about the whole object, such as finding surface area, identifying symmetry, or predicting how the object would look after rotation.

Makamure and Jojo (2021) also confirmed that students frequently struggle to understand spatial relationships between 2D and 3D representations. For example, when presented with a 2D cube image, students may find predicting how it would appear after a 90-degree rotation challenging. All these indicate a broader issue with spatial visualization and the ability to manipulate mental images of geometric objects.

Triutami et al. (2021) investigated students' mental ability to construct and deconstruct geometric figures. Their findings revealed that students often struggle to form a 3D object from a 2D object or to visualize how a 3D object would look when unfolded into a flat layout. These difficulties are particularly pronounced at the lower secondary level, where spatial reasoning skills are still developing.

To the above discussion, visual-spatial skills are essential in learning geometry, enabling students to mentally manipulate shapes and interpret spatial relationships (Clements, 1992). Students struggle with tasks related to visual-spatial skills, such as transforming between 2D and 3D representations and visualizing rotations (Van Garderen, 2006; Zhang et al., 2014). Difficulties include imagining hidden parts of objects (Wahab et al., 2014), tracking orientation changes (Kalun et al., 2020), and constructing or unfolding 3D shapes (Makamure & Jojo, 2021; Triutami et al., 2021). These challenges limit students' CU and solving geometry problems, highlighting the need to strengthen visual-spatial reasoning in geometry instruction. All these difficulties mostly occur in lower secondary education (Clements, 1992; Kalun et al., 2020; Makamure & Jojo, 2021; Triutami et al., 2021; Van Garderen, 2006; Wahab et al., 2014; Zhang et al., 2014). Thus, visual-spatial skills in this study refer to the ability to perceive, interpret, and mentally manipulate visual information related to space and geometric shapes.

### **2.2.1.3 Visual Representation**

Žakelj and Klancar (2022) highlighted the fundamental role of VR in helping students understand abstract geometric concepts. They argued that VR is a tool for illustration and a cognitive process that allows learners to mentally manipulate geometric figures and relationships. This mental manipulation leads to a deeper understanding of spatial relationships and geometric shapes. For example, when students visualize transformations such as rotations or reflections, they engage in abstract reasoning supported by visual thinking. They also point out that many

students, especially at the upper secondary level, face challenges in constructing accurate geometrical figures to clarify problems. Students often fail to use visualization as a bridge to link concrete experiences with abstract mathematical thinking.

López and Pintó (2017) focused on students' difficulty integrating multiple representations, especially when solving geometric problems. They found that students frequently struggle to coordinate visual, symbolic, and verbal forms of representation. Those difficulties included the tasks related to explaining geometric relationships through geometrical shapes while simultaneously using mathematical language or symbolic notation. For instance, a student might draw a triangle but fail to verbally describe the properties of its angles or sides or be unable to connect the shapes to a formal geometric proof. They also emphasized that such integration is critical for meaningful learning in geometry, yet it remains a persistent challenge for learners.

Saifiyah and Retnawati (2019) also emphasized students' struggles with representation, particularly in expressing geometric reasoning through visual and symbolic means. They found that students often lack the skills to construct visual aids that reflect the relationships between geometric figures, theorems, and verbal reasoning. These include difficulties translating a word problem into an accurate shape or using figures to justify mathematical conclusions. Their research supports that students rely on rote procedures without strong representational skills rather than engaging in deeper conceptual reasoning. They also note that such challenges are more prevalent in upper secondary education levels, where the complexity of geometric reasoning increases.

Zhang et al. (2014) provided further insight by discussing how students fail to construct and use various visual aids effectively. They explain that students need to be able to move between representations, such as drawing shapes, plotting coordinate systems, and using appropriate symbols to understand geometric relationships fully. For example, sketching a parallelogram and simultaneously understanding its properties through algebraic expressions or verbal justifications is a sign of high-level visual-spatial reasoning. They stressed that this coordination is crucial for developing problem-solving skills, yet many students cannot perform these tasks seamlessly.

Through the above studies, the fundamental concept of VR can help students understand abstract concepts and concretely support spatial reasoning. VR is essential in geometry learning, as it leads students to manipulate geometric figures and relationships mentally, leading to a deeper understanding of the concept involved (Žakelj & Klancar, 2022). This VR is related to recognition.

The VR aspect that students face difficulty with is drawing geometrical figures to clarify the problem and integrate multiple representations, such as symbolic notation and verbal explanation, including expressing reasoning and explaining the relationships between geometric shapes, theorems, and verbal expressions (López & Pintó, 2017; Saifiyah & Retnawati, 2019; Zhang et al., 2014). This VR is related to reasoning.

Students struggle to create geometrical drawings to clarify the problems and integrate multiple representations, which involves creating and using various forms of visual aids to understand and communicate geometric concepts; for instance, students may sketch shapes, angles, and figures to visualize and solve geometric problems; use the coordinate system to plot points, lines, and curves to understand relationships between geometric entities; and employ symbols and notations to represent geometric properties and relationships (Žakelj & Klancar, 2022; Zhang et al., 2014). This VR is related to an external representation

These discussions summarize VR into three points: recognition, mental operation, and external representation. All these difficulties mostly occur in upper secondary education. To meet the research purpose, ***VR is defined as the student's ability: (1) to identify and interpret visual information, including visual, symbolic, and verbal representations (Recognition); (2) to transform and coordinate different types of picture representations in mind (Mental Operation); and (3) to create geometrical drawings to clarify the problem (External Representation).*** Next, we will discuss the relationship between the three aspects of visualization.

### 2.2.1.4 Relationship Between Visual Perception, Visual-Spatial Skills, and Visual Representation

All three aspects, visual perception, visual-spatial skills, and VR, are essential in solving geometry problems. Visual perception enables students to recognize shapes, visual-spatial skills help them mentally manipulate those shapes, and VR allows them to express their understanding concretely through drawings or symbols (Baluoti et al., 2012; Clements, 1992; Gal & Linchevski, 2010; Richmond & Holland, 2010; Saifiyah & Retnawati, 2019; Sieng & Thai, 2024a; Triutami et al., 2021; Žakelj & Klancar, 2022; Zhang et al., 2014). For example, a student identifies a triangle among other polygons with the following figures.

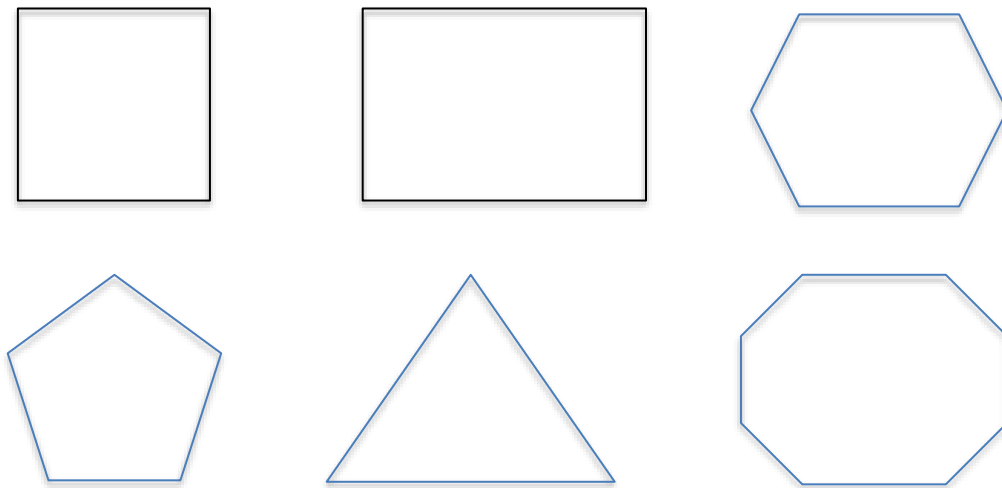


Figure 4. An example of the relation between visual perception, visual-spatial skills, and VR in identifying a triangle among other polygons.

The process of identifying a triangle among other polygons begins with visual perception, where the student detects and distinguishes shapes based on geometric shapes such as the number of sides and angles; this is followed by visual-spatial skill, which allows for mental manipulation of the shapes, such as rotation or reflection to confirm whether a shape with three sides is still a triangle in different orientations; finally, VR enables the learner to express, verify, or communicate the identified triangle through drawing, marking, or comparing it with a mental or external model, thereby completing the reasoning process from observation to conclusion.

These aspects differ in their specific focus in terms of their implementation stages. Visual perception involves recognizing external shapes. Visual-spatial skills involve students in manipulating objects and abstract shapes mentally. Students need

to develop to a higher level to conduct VR, which consists of using the relevant skills to perform picture representation through drawing or diagrams of the shapes (Clements et al., 1992; Žakelj & Klancar, 2022).

Therefore, visual perception, visual-spatial skills, and VR are distinct yet interrelated cognitive processes for geometry. Understanding how they differ and work together can help teachers design more effective teaching strategies to address students' difficulties in geometry (Clements et al., 1992; Gal & Linchevski, 2010; Žakelj & Klancar, 2022). They are related to students' learning and solving geometry problems. For example, to solve a geometric problem, a student might first use visual perception to identify the shapes involved, then apply the visual-spatial skill to manipulate these shapes mentally, and finally use VR to draw shapes or figures that aid in solving geometry problems (Gal & Linchevski, 2010; Jones & Tzekaki, 2016). The different education stages in general education implement the three visualization aspects as required. In primary education, the focus is on visual perception; this leads to visual-spatial skills in lower secondary education, and finally, the emphasis shifts to VR in upper secondary education (Sieng, 2024; Sieng & Thai, 2024b).

Moreover, each concept has a distinct function: perception is about recognizing, spatial ability is about mental manipulation, and representation is about visual expression. However, students should build their knowledge and connect each aspect to the other by understanding that perception supports spatial knowledge, which in turn enhances representation (Azizah et al., 2021; Cornsweet, 2012; Crissey, 2023; Khalil et al., 2024; Makamure & Jojo, 2021; Yue, 2008; Žakelj & Klancar, 2022). Table 9 summarizes the differences and relations between visual perception, visual-spatial skills, and VR.

*Table 9. Differences and Relations Among Visual Perception, Visual-Spatial Skills, and VR*

<b>Aspect</b>	<b>Visual perception</b>	<b>Visual-spatial skills</b>	<b>VR</b>
<b>Focus</b>	. Recognize shapes, colors, and spatial features.	. Understand positions, movement, and transformation of objects.	. Create and interpret visual models.
<b>Cognitive process</b>	. Recognize and interpret information from the eyes.	. Use mental manipulation of objects in space.	. Use external expression of geometric ideas through visual objects.
<b>Example</b>	. Identify a triangle among other polygons	. Use mental manipulation of the shape to see the triangle in different orientations.	. Confirm, express, verify, and identify a triangle through drawing by completing the process from observation to conclusion.
<b>Relationship</b>	. Provide the foundation for recognizing objects.	. Use perception to understand spatial features and aid representation by supporting visualization.	. Rely on perception for accurate depiction and spatial ability to construct meaningful visuals.
<b>Differences</b>	. Interpret visual stimuli from the environment.	. Use mental manipulation and visualize spatial relationships.	. Use diagrams, graphs, and symbols to express information.

Source: The author developed the table based on a literature review.

### **2.2.2 Relationship Between CU and VR**

As stated in the visualization section, VR is one of the three important points that help students face difficulties in geometry in upper secondary education, which

leads to a lack of CU. In that sense, VR and CU are closely connected to how students learn and make sense of mathematics, especially geometry. Both are related to each other. VR acts as a cognitive tool that supports the development of CU by enabling learners to mentally manipulate geometric figures, such as rotations or reflections, thus facilitating the comprehension of abstract geometric relationships (Žakelj & Klancar, 2022). This mental operation allows students to internalize and make sense of geometric properties rather than memorizing procedures, leading to a deeper understanding of why mathematical rules work. For example, visualizing how the interior angles of a triangle sum to 180 degrees helps students grasp this property conceptually rather than accepting it as a fixed rule (Leinwand et al., 2014).

VR is crucial in supporting reasoning and justification, which are core components of CU. When students use visual tools, such as sketches, diagrams, or dynamic software, like GeoGebra, they can explore, test, and justify geometric relationships, enhancing their ability to explain why a specific mathematical procedure is valid (Fujita & Jones, 2007; Irshid et al., 2023). Without well-developed visual skills, students often find it difficult to justify their reasoning, indicating gaps in their conceptual grasp (Saifiyah & Retnawati, 2019). In addition, the ability to coordinate multiple forms of representation, such as visual, symbolic, and verbal, is crucial for building a coherent understanding of geometry. Students who can shift between these forms demonstrate what Rupnow and Fukawa-Connelly (2024) describe as representational fluency, which is key to meaningful learning and problem-solving.

Furthermore, for teachers, visualization should be seen as a supportive illustration and an integral part of instruction. According to Arcavi (2003) and Presmeg (2020), visual thinking is central to teaching and learning geometry, as it enables teachers and students to explore relationships, identify properties, and justify conclusions. Teachers with strong CU can use various visual tools, such as diagrams and digital software, to support students' reasoning and help bridge the gap between concrete experiences and abstract ideas (Clements, 1992; Irshid et al., 2023).

Many students face challenges in linking VR with CU. Researchers show that students may construct geometric figures correctly but struggle to interpret or explain their meaning using appropriate mathematical expressions or symbolic notation

(López & Pintó, 2017; Zhang et al., 2014). These difficulties suggest the need for instructional approaches that simultaneously develop CU and VR skills.

Therefore, VR and CU are deeply interrelated in geometry learning. VR helps students recognize, interpret, and manipulate geometric figures and properties, while CU enables them to justify procedures and solve problems meaningfully. Thus, developing both abilities in tandem is essential for fostering a deep, integrated understanding of geometry (Gurmu et al., 2024; Rupnow & Fukawa-Connelly, 2024).

### **2.2.3 Relationship Between CU and Visualization**

CU in geometry is significantly enhanced through the use of visualization. Engaging with dynamic representations, geometric constructions, and transformation processes enables students to more effectively comprehend geometric properties, relationships, and proofs. Visualization is a cognitive tool that helps students make abstract concepts more accessible and meaningful. Students who possess a strong CU have adequate visualization. They can visualize more accurately when they grasp the fundamental properties of shapes and transformations. For instance, a student who understands that the sum of angles in a triangle is always 180 degrees can visualize different triangles while keeping this property in mind (Duval, 1998). This understanding enhances the ability to visualize and interpret diagrams accurately, helping to avoid misinterpretations.

As discussed in the CU section, we concluded that visualization and CU are related. Visualization enhances CU to assist students in making abstract geometric concepts more concrete by allowing them to see and critically use shapes, relationships, and transformations. For example, when students visualize geometric transformations like rotations or reflections, they use the image and deepen their understanding of the properties involved, such as congruence and symmetry (Clements, 1992). Visualization connects abstract concepts with tangible experiences, facilitating the internalization and understanding of geometric ideas (Arcavi, 2003).

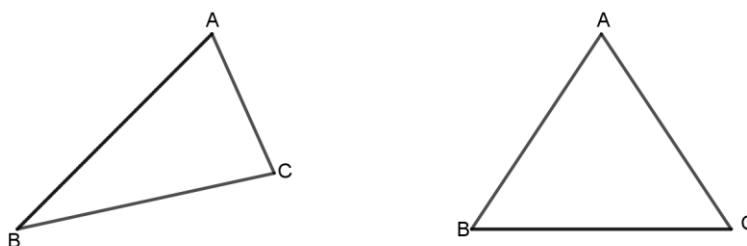
Scholars highlighted that students' poor visualization skills could lead to poor CU (Arcavi, 2003; Hegarty & Kozhevnikov, 1999; Presmeg, 2020). Visualization skills encompass the capacity to mentally represent, interpret, and manipulate spatial and visual information, serving as a foundation for meaningful learning. When these skills are underdeveloped, students may struggle to form coherent connections between

abstract ideas and concrete representations, leading to superficial or incomplete understanding. For instance, Arcavi (2003) emphasized the essential role of visualization as a cognitive tool in the learning process. He suggests that visualization facilitates the transition from concrete objects to abstract reasoning by providing learners with accessible mental images. Similarly, Hegarty and Kozhevnikov (1999) highlighted that challenges in visualization could create cognitive barriers, making it difficult for students to engage with complex tasks or process new information effectively.

Presmeg (2020) argued that weak visualization abilities may prevent students from understanding the relationships between mathematical concepts, thereby limiting their capacity for deep learning. The students' ability to visualize the relationships helps them understand the concept rather than just memorizing rules and formulas. Students who lack visualization skills can limit their ability to solve problems and identify connections between different concepts.

Educators must adopt strategies that support the development of visualization skills. The use of visual aids, such as diagrams, animations, and dynamic geometry software, like GeoGebra, has been shown to enhance students' spatial reasoning and engagement with abstract concepts (Gurmu et al., 2024; Salami & Spangenberg, 2024; Siswanto et al., 2024; Zhang et al., 2023). Furthermore, targeted interventions, including guided practice and spatial reasoning exercises, can address individual student needs, fostering visualization development and CU skills.

In geometry, after finding out the characteristics of shapes, it is crucial to represent them as properties of the shapes. For example, a triangle is a plane shape that has three sides. Among the triangles, an isosceles triangle has two sides of the same length. They may be represented as the diagrams below or triangle ABC and  $AB=AC$ .



*Figure 5. Properties of triangles.*

Furthermore, reasoning plays an explicit role. Argumentation and proof are very critical. Again, visualization plays a central role in them. For example, a student learning about the Pythagorean Theorem can visualize squares constructed on the sides of a right triangle, helping them understand the relationship  $a^2 + b^2 = c^2$  rather than merely memorizing it (Battista, 1990). This visualization solidifies their conceptual grasp by linking symbolic, visual, and spatial representations (Irshid et al., 2023).

These relationships show that visualization and CU are interconnected and correspond with each other. Figure 5 illustrates the conceptual framework of the relationship between VR and CU.

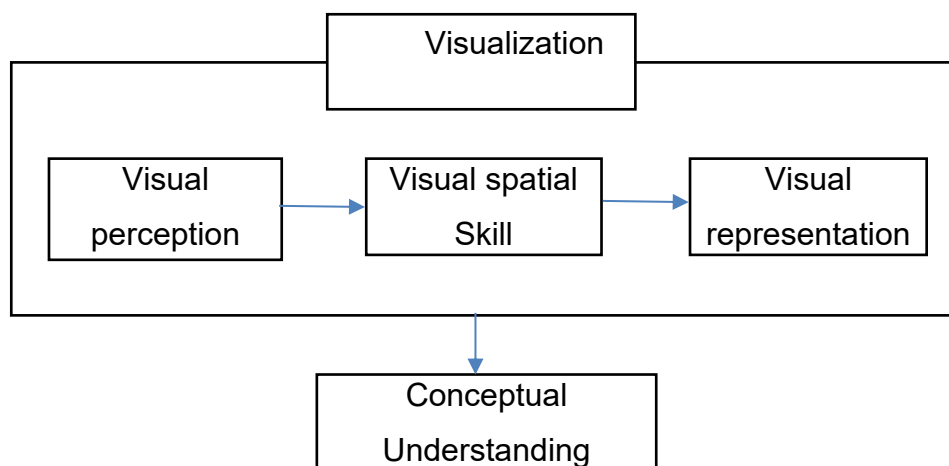


Figure 6. Conceptual framework of the relationship between visualization and CU

Source: The author developed based on the literature review.

This figure illustrates the relationships between visualization and CU. Visualization involves three interconnected components: visual perception, visual-spatial skills, and VR. These components work in sequence, where visual perception supports visual-spatial skills, enabling VR. These skills extend into VR, in which concepts are externalized through drawing or digital tools, and CU. In summary, the illustrated process demonstrates how perception and spatial skills work together to improve students' abilities to represent and understand geometry concepts. According to the previous discussion, visualization and CU are interrelated. Thus, students who lack either of the two tend to have low achievement scores.

## **2.3 APPROACHES IN GEOMETRY LEARNING AND DYNAMIC GEOMETRY SOFTWARE**

### **2.3.1 Approaches in Geometry Learning**

Numerous approaches are used to overcome students' difficulties in learning geometry, such as reforming the educational policy, analyzing the curriculum, strengthening school management, building capacity among personnel, improving teachers' knowledge of geometry, developing effective teaching and learning materials, improving teaching methods, using DGS, and encouraging students to learn geometry (Sieng & Thai, 2024b). Among them, the utilization of DGS is one of the possible approaches that can help students improve their abilities in solving geometry problems and overcome VR and CU (Arbain & Shukor, 2015; Azizah et al., 2021; Hasibuan et al., 2019; Hohenwarter et al., 2010; Kholid, 2022; Purevdorj, 2016; Sieng & Thai, 2024b; Teodorović, 2011; Ulandari et al., 2019; Wang et al., 2023).

DGS is a computer application designed for the interactive creation and manipulation of geometric figures. It allows users to work with geometric objects like points, lines, and circles, maintaining their dependencies. When users move parts of a construction, the program adjusts other parts to preserve geometric constraints (Bantchev, 2010; Chan & Leung, 2014; Hohenwarter et al., 2010; Sieng & Thai, 2024b). Unlike simple drawing programs, DGS can visualize and manipulate abstract geometric figures. It is used to visualize complex geometric data, compare the actual drawing object as a model, combine geometric shapes and algebra, perform calculations, test hypotheses, and visualize and create precise illustrations for documents or the web. Typically, DGS starts its operation by creating independent objects (e.g., points) and proceeds by constructing dependent ones. In this sense, DGS allows users to dynamically create and modify geometrical figures. DGS instruction can enhance mathematics education and improve students' mathematical achievement (Bantchev, 2010; Chan & Leung, 2014; Hohenwarter et al., 2010).

Another important approach is that the curriculum is one of the most influential factors that can help students overcome the difficulty in learning geometry because it is the starting point for teaching and learning activities (UNESCO, 2023). It should be well-organized and coherent (Cuoco & McCallum, 2018; Schmidt et al., 2002). Curriculum coherence is the most dominant predictor for student achievement

because it refers to the connection and logical progression of content, skills, and learning outcomes within a curriculum framework, syllabus, textbook, and assessment (Cuoco & McCallum, 2018; Prawat & Schmidt, 2006; Schmidt et al., 2002; Sieng & Baba, 2024). Next, we discuss the relationship between DGS, CU, and curriculum coherence.

### **2.3.1.1 Relationship Between DGS and CU**

DGS plays a significant role in supporting and enhancing students' CU of geometry. CU refers to the deep comprehension of mathematical principles, relationships, and reasoning processes that enable learners to make connections, justify procedures, and apply knowledge flexibly across different contexts (Howe, 1999; Leinwand et al., 2014; Rupnow & Fukawa-Connelly, 2024). DGS provides an interactive and visual environment that aligns well with these cognitive demands by allowing learners to dynamically construct, manipulate, and explore geometric figures.

One of the key contributions of DGS to CU is its ability to visualize abstract geometric concepts in a concrete and manipulable form. As students create geometric constructions, DGS maintains the logical dependencies between elements, enabling real-time feedback when figures are altered. This immediate visual response reinforces the relationships between geometric properties, such as congruence, symmetry, and angle measures, helping students internalize these concepts more deeply (Bantchev, 2010; Hohenwarter et al., 2010). For example, when a student drags a vertex of a triangle and observes that the sum of the internal angles remains  $180^\circ$ , they engage in conceptual reasoning supported by visual interaction.

Moreover, DGS promotes reasoning and justification, which are fundamental components of CU. The software allows learners to test hypotheses, explore invariance, and observe the consequences of changes within geometric constructions. This process encourages students to move beyond procedural knowledge and develop the ability to explain why geometric rules hold true (Fujita et al., 2020; Irshid et al., 2023). Students learn to construct mathematical arguments grounded in visual evidence by engaging in exploration and conjecture, strengthening their reasoning skills.

Additionally, DGS facilitates the integration of multiple visual, symbolic, and verbal representations, which is essential for building CU. As students interact with

geometric figures, they can simultaneously observe algebraic relationships, use measurement tools, and describe properties verbally. This multimodal engagement supports the coordination of representations and enhances representational fluency (López & Pintó, 2017; Rupnow & Fukawa-Connelly, 2024). Through this process, students can better understand the connections between geometric models and the symbolic expression of mathematics.

Research also indicates that using DGS in instructional settings improves student outcomes in mathematics. Studies by Chan and Leung (2014) and Hohenwarter et al. (2010) have shown that DGS fosters student engagement, enhances geometric thinking, and improves performance by making abstract ideas accessible and interactive. By allowing students to construct knowledge through exploration and manipulation, DGS aligns with constructivist principles of learning, which emphasize understanding over memorization.

Therefore, integrating DGS significantly contributes to the development of CU in geometry. It enhances students' ability to visualize, reason, and justify geometric relationships while supporting the flexible use of multiple representations. So, DGS is not merely a technological tool but a pedagogical resource that facilitates more profound and meaningful learning of geometric concepts. Thus, in this study, the possibility of using DGS is identified.

### **2.3.1.2 Relationship Between DGS and Curriculum Coherence**

DGS contributes meaningfully to promoting curriculum coherence in mathematics education, particularly in geometry. Curriculum coherence refers to the logical sequencing and alignment of content, skills, instructional materials, and assessments, ensuring that learning progresses meaningfully and systematically within and across the grade level (Cuoco & McCallum, 2018; Prawat & Schmidt, 2006; Schmidt et al., 2002; Schmidt et al., 2005a). When technology such as DGS is aligned with curricular goals and effectively integrated into instructional practices, it strengthens the internal coherence of the curriculum by reinforcing connections between concepts, facilitating continuity across grade levels, and supporting consistent learning experiences.

DGS supports curriculum coherence by enabling the dynamic visualization of geometric principles introduced progressively within a curriculum framework. For

instance, geometric transformations, congruence, similarity, and construction topics that are usually introduced at different stages can be revisited, explored, and reinforced through DGS in a connected and cumulative manner. As students manipulate and observe geometric figures in real time, they build on prior knowledge while deepening their understanding of new content. This directly supports the vertical alignment of the curriculum, which is essential for coherent content progression (Schmidt et al., 2002; Sieng & Baba, 2024).

Moreover, DGS enhances the horizontal coherence of the curriculum by bridging the gap between learning objectives, instructional activities, and assessment tasks. Teachers can use DGS to visualize abstract geometric concepts and connect them to multiple representations (Bantchev, 2010; Chan & Leung, 2014). This coherence ensures that the curriculum is not fragmented or overly procedural but promotes deep CU consistent with curriculum goals (Cuoco & McCallum, 2018).

In addition, DGS aligns well with assessment practices that reflect curricular expectations. For example, tasks that require reasoning, justification, and exploration of key elements of curriculum coherence can be effectively supported by DGS-based activities. When students construct geometric arguments or test conjectures using DGS, they engage in higher-order thinking aligned with the intended learning outcomes outlined in coherent curricula (Hohenwarter et al., 2010; Irshid et al., 2023).

However, to realize the full benefits of DGS in promoting curriculum coherence, its use must be systematically integrated into the curriculum design, teacher training, and instructional planning (Albeshree et al., 2022; Salami & Spangenberg, 2024). If implemented in isolation or without alignment with curriculum goals, DGS might not contribute to coherence and may introduce disconnection or confusion. Hence, when embedded within a well-structured curriculum framework, DGS acts as a powerful pedagogical tool that reinforces the connectedness, clarity, and progression of geometric content.

Therefore, integrating DGS into teaching and learning supports curriculum coherence by facilitating content, pedagogy, and assessment alignment. It reinforces vertical and horizontal coherence, promotes cumulative learning experiences, and enables students to engage dynamically and meaningfully with geometry. As such, DGS is a technological innovation and a key instrument in realizing the goals of a

coherent and high-quality mathematics curriculum. So, there is a strong relationship between DGS and curriculum coherence. Thus, this study analyzes the curriculum from a coherent perspective.

### 2.3.2 Type and Features of DGS

Three main types of DGS packages were used for different purposes: to present objects as two-dimensional (2D), three-dimensional (3D), or both (2D and 3D) to support teaching and learning geometry. Table 10 shows their classification.

*Table 10. Classification of the use of DGS in different representations*

Category	Software
2D	Desmos, Geometry Expressions, Cinderella, Graphing Calculator, Sketchpad
3D	Cabri 3D, 3D Calculator, Archimedes Geo 3D, Geometrix
Both	GeoGebra

Source: The author developed the table based on a literature review.

The DGS packages mentioned in Table 10 are available online (designed to run directly in a browser without installation), offline (require installation and work on local devices), and both (support both web-based and downloadable versions for flexibility). Their availabilities are shown in Table 11.

*Table 11. DGS packages are available online, offline, or both*

Category	Software
Online	Desmos, Cinderella (via CindyJS), GeoGebra (web-based version), 3D Calculator
Offline	Cabri 3D, Graphing Calculator, Geometrix, Sketchpad, Archimedes Geo 3D
Both	GeoGebra, Cinderella, Geometry Expressions

Source: The author developed the table based on a literature review.

It is important to note that the online DGS packages are open-source, while the offline ones are paid, and some are used in both open-source and paid versions, depending on the specific purpose.

According to MoEYS (2020b), nine DGS packages in Cambodia's education system, mentioned in section 1.1.5, are utilized to support mathematics teaching and learning. Among these software packages, as well as those mentioned in Table 10, four were possible to integrate into the geometry domain: GeoGebra, Desmos, 3D Calculator, and Graphic Cals, and they were cascaded training to some schoolteachers for implementation throughout the country to support teaching and learning mathematics, specifically to enhance teaching and learning geometry at upper secondary schools in Cambodia (MoEYS, 2020b).

It is essential to evaluate various features of DGS programs before selecting one to ensure it meets specific needs and expectations. The features of DGS include graphical representations of geometry on the screen, exploration of geometric properties and hypothesis testing, visualization of complex data, geometric reasoning, illustrations in document preparation, illustrations for the Web, and libraries for geometric programming (Gökçe, 2022; Kasik, 2019). A DGS package may offer many features; however, only some may be available to the user. Moreover, some specific DGS features, such as calculations, loci, animations, and proofs, support geometry learning, while other features can be used for additional support to mathematics education in general, such as macros, scripting, assignments, LaTeX exporting, and multilingual options (Bantchev, 2010; Chan & Leung, 2014; Hohenwarter et al., 2010). A comparison of the software packages was conducted to identify which of the features is most commonly utilized in supporting teaching and learning geometry.

### **2.3.3 Comparison of DGS**

To evaluate the most commonly used features for improving teaching and learning geometry, DGS packages mentioned in the previous section were compared. Among them, four are compatible with geometry content in the Cambodian context, while the others are applicable to different contents (MoEYS, 2020b). The comparison focuses on calculations, loci, animations, and proofs, which are commonly used to visualize geometrical shapes and algebra. The packages also have additional features in other areas of mathematics, such as algebra, calculus, statistics, and graphing, with strong community support from software developers. The comparison is shown in Table 12.

Table 12. Comparison of the dynamic geometry software packages used to visualize geometry shapes and algebra.

Software	Calculations	Loci	Animations	Proofs	Additional features
<b>Cabri 3D</b>	✓	✓ (3D loci)	✓	✓ (limited)	. 3D visualization for geometry and spatial reasoning.
<b>Geometry Expressions</b>	✓	✓	X	✓	. Integrates geometry with algebra and provides symbolic algebraic representations.
<b>Cinderella</b>	✓	✓	✓	✓ (probabilities)	. Includes physics simulations, interactive web exports, and dynamic geometric proofs.
<b>Desmos</b>	✓	✓	✓ (limited)	x	. User-friendly graphing interface; limited to educational uses and graphing.
<b>GeoGebra</b>	✓	✓	✓	✓	. Integration support with 2D and 3D visualization, including probability, algebra, statistics, calculus modules, and graphing; strong community support from software developers.
<b>3D Calculator</b>	✓	✓	✓	x	. Focuses on 3D plotting and dynamic visualization; limited proof capabilities.

<b>Graphic Calculator</b>	✓	x	x	x	. A simple calculator for basic graphing; lacks advanced features.
<b>Geometricx</b>	✓	✓	✓	✓ (limited proof capabilities)	. Emphasizes geometry but lacks scripting or proof capabilities.
<b>Archimedes Geo 3D</b>	✓	✓	✓	✓ (limited)	. Strong 3D geometry tool for visualizing complex constructions.
<b>Sketchpad</b>	✓	✓	✓	✓ (limited)	. Dynamic geometry software focused on educational uses.

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Source: The author developed the table based on a literature review.

Four dynamic geometry software (DGS) packages have been utilized to support the teaching and learning of geometry at the upper secondary level in Cambodia. GeoGebra was identified as the most suitable due to its integrated geometric and algebraic problem-solving capabilities. Specifically, Desmos, 3D Calculator, and Graphic Calculator are built into the GeoGebra software as toolbars, making it a comprehensive tool. Moreover, GeoGebra supports 2D and 3D visualization and includes modules for probability, algebra, statistics, calculus, and graphing, all enhanced by strong community support from software developers (Hohenwarter, 2004). Therefore, GeoGebra was selected for investigation in this study.

### 2.3.4 Description of GeoGebra Software

GeoGebra software was developed by Markus Hohenwarter in 2001/2002 as part of his master's thesis at the University of Salzburg, Austria. Supported by a DOC scholarship from the Austrian Academy of Sciences, he continued developing it during his PhD (Hohenwarter & Fuchs, 2004). GeoGebra has won multiple international awards and has been translated into over 25 languages. Since 2006, it has been supported by the Austrian Ministry of Education to ensure its free availability for

educational use. In 2006, its development extended to the U.S., continuing at Florida Atlantic University through a National Science Foundation project.

The GeoGebra interface is easy for users to see and explore as a dynamic visualization. It provides many geometrical tools for users to draw geometrical shapes, view 2D and 3D graphics, and access algebraic input bars; moreover, community software developers are available to assist users in viewing the concept of geometrical shapes in relation to visualization. The GeoGebra interface is shown in Figure 7 below.

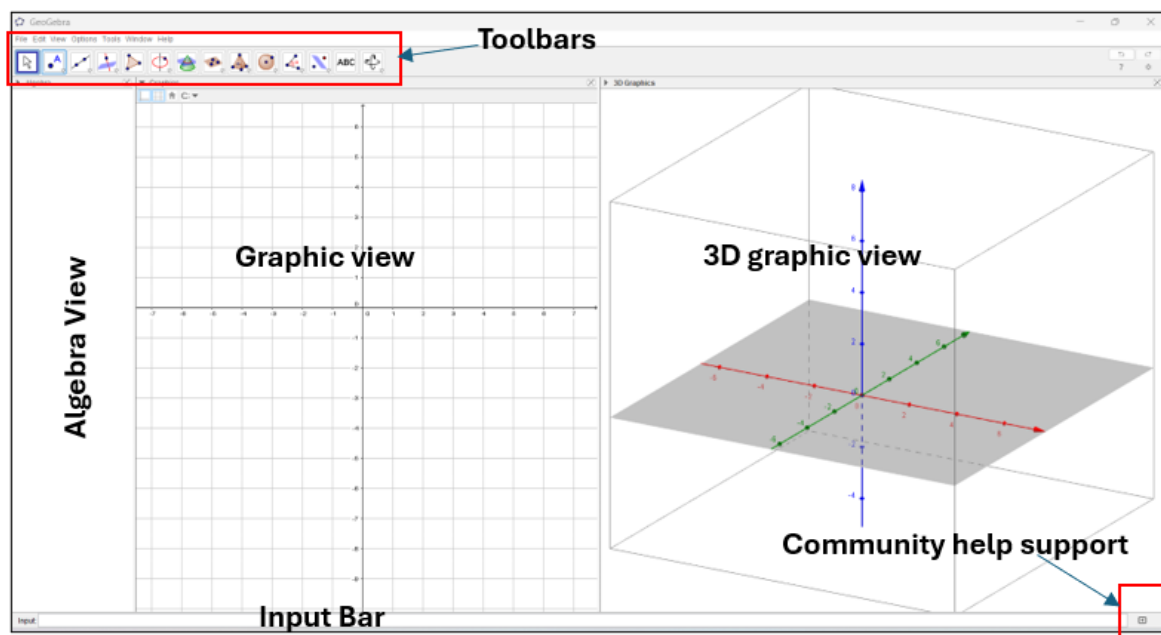


Figure 7. GeoGebra interface capture

Source: The author developed the figure based on the GeoGebra manual.

GeoGebra provides a significantly enhanced VR and CU by making mathematical ideas tangible, interactive, and accessible. Its ability to combine algebraic procedures and graphics representations allows students to use their visualization skills to enhance their understanding of more profound mathematical procedures and gain a more holistic grasp of mathematical concepts (Liu, 2021; Orozco, 2017). Figure 8 shows how GeoGebra can help students in achieving VR and CU.

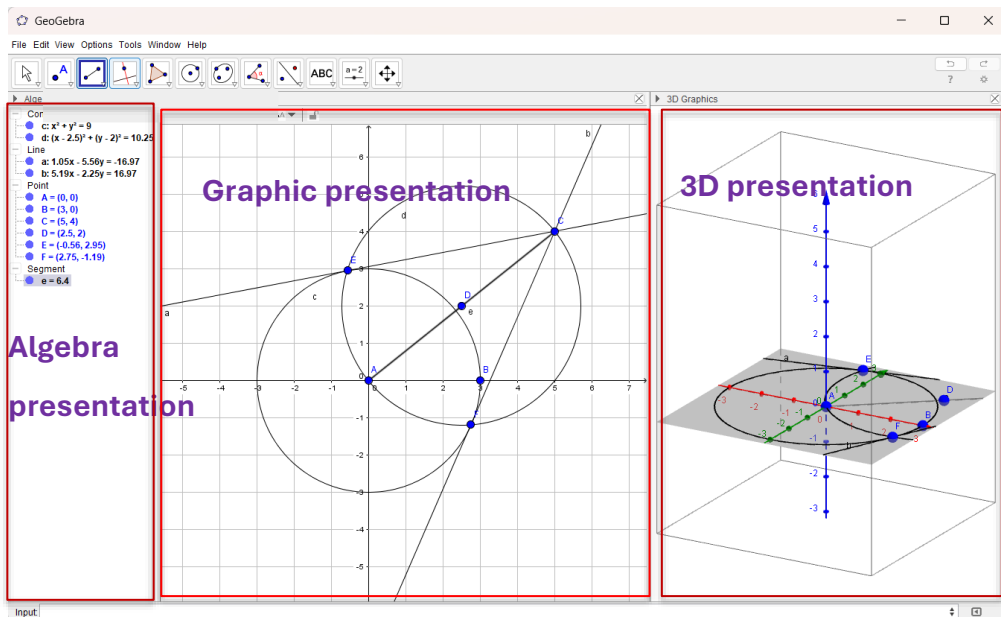


Figure 8. The GeoGebra interface presents a VR and enables CU.

Researchers indicate that integrating GeoGebra software enhances student performance and encourages active learning (Bhagat et al., 2015; Shadaan & Leong, 2013). It can overcome students' challenges in VR and improve CU through dynamic, interactive visualizations that address abstract mathematical concepts with algebraic procedures and graphical representations. It allows users to manipulate geometric objects, algebraic expressions, and graphs in real time, fostering a deeper understanding of how changes in one representation affect the others. For instance, a study by Bhagat et al. (2015) discovered that GeoGebra significantly enhanced students' performance in geometry by providing a dynamic environment for exploring mathematical properties. Similarly, it has been suggested that GeoGebra encourages active learning and aids in the construction of mental models, which supports creative ideas.

GeoGebra software was introduced in Cambodia's mathematics education system in 2014 by GeoGebra International Institute members. Moreover, the Cambodian community of mathematicians continues to welcome mathematics teachers and enables networking through its annual meeting. One such meeting was held in 2014 in Siem Reap province. In 2016, MoEYS organized a national mathematics conference in Kompot province. The conference aimed to enhance the quality and effectiveness of mathematics education in Cambodia, with many objectives, including introducing GeoGebra software in the upper secondary school








curriculum. In 2017, MoEYS developed a GeoGebra guidebook in the Khmer language for teachers to further their learning with this software. However, it was not implemented in the classroom for various reasons: lack of ICT infrastructure, lack of ICT laboratories, and the teachers' inability to use the software (Pang et al., 2022).

In 2020, a guidebook on using tablets to support the teaching of science and mathematics was created with the assistance of the second upper secondary education sector development program (USESDP II). It provided cascade training to some SRS teachers who teach science, mathematics, and ICT on integrating tools into teaching and learning in their subjects (MoEYS, 2020b). However, the technical department or MoEYS have not yet created an assessment process to measure the effectiveness of using these tools.

### **2.3.5 Features and Strengths of GeoGebra Software**

GeoGebra is dynamic mathematics software that combines geometry, algebra, calculus, and other mathematical tools. Its features provide dynamic tools for teaching and learning in various educational contexts. It is one of the programs that can help teachers and students explore mathematical knowledge; it includes many features that allow users to calculate, demonstrate, and visualize facts (Hohenwarter, 2004). Table 13 presents the features of GeoGebra software.

Table 13. The features of GeoGebra software.

apps / features	 Scientific	 Graphing	 Geometry	 3D	 CAS	 Suite	 Classic
Numeric calculations	✓	✓	✓	✓	✓	✓	✓
Function operations	✓	✓	✓	✓	✓	✓	✓
Fraction operations	✓	✓	✓	✓	✓	✓	✓
Graphing		✓	✓	✓	✓	✓	✓
Sliders		✓	✓	✓	✓	✓	✓
Vectors & matrices		✓	✓	✓	✓	✓	✓
Table of values		✓			✓	✓	✓
Geometric constructions			✓	✓	✓	✓	✓
3D graphing				✓		✓	✓
Probability Calculator						✓	✓
Derivatives & integrals				✓	✓	✓	✓
Equation solving				✓	✓	✓	✓
Symbolic calculations				✓	✓	✓	✓
Spreadsheet							✓

Source: GeoGebra manual

GeoGebra software has provided many possible geometrical applications: scientific, graphing, geometry, 3D, CAS, Suite, and Classic. All the features integrated into this application are user-friendly (Hohenwarter et al., 2009). Therefore, GeoGebra can help students explore many features to improve their geometrical concepts and skills. More importantly, the software is free.

GeoGebra software can be used at all education levels to improve students' learning achievement and overcome their learning difficulties (Gurmu et al., 2024; Hohenwarter et al., 2009; Mullis et al., 2016; Saidu & Rabi, 2024; Salami & Spangenberg, 2024; Uwurukundo et al., 2020). The scholars discussed the trends in integrating GeoGebra software, which is possible from primary education to upper secondary education for general education (see Table 14).

Table 14. The trends in integrating GeoGebra software by education level

Education level	Countries	Contents	Authors
Primary education	Malaysia, Hungary, Bosnia & Herzegovina, Slovakia, Lithuania, Turkey, Indonesia, Brazil	Basic geometry (shape and space), Number concepts, Measurement, Data handling, Patterns, and Algebra	(Mazian, 2019; Mullis, 2016)
Lower secondary education	Bhutan, United States, Germany, United Kingdom, France, Finland, Australia, Japan, South Korea, Singapore	Dynamic geometry, Mathematics, and Science	(Tomas, 2023; Mullis, 2016)
Upper secondary education	United States, Germany, South Korea, Australia, Canada, Japan, Spain, Nepal, and South Africa	Euclidian geometry, Geometry concepts, Geometrica 1 reflection, Learning coordinate geometry, Learning circles, Analytical geometry, Integration, Limits, Continuity, Derivatives, Linear functions, Linear algebra, Trigonometric functions	(Uwurukundo, 2020)

Sources: Developed by the author based on a literature review

Table 14 illustrates that GeoGebra can be used at all education levels to support teaching and learning in the classroom, that is, primary, secondary, and upper secondary education.

### 2.3.6 Global Trends in Integrating GeoGebra Software into Upper Secondary Education

GeoGebra software has been used in various countries, particularly in upper secondary schools, where mathematics has become more complex. Students and teachers need to use IT, especially GeoGebra software, to assist in their teaching and learning and solve complex problems. Many countries have already used GeoGebra software to enhance students' learning achievements, particularly in upper secondary education (Table 15) (Em & Roman, 2020; Herceg & Herceg, 2010; Martinez, 2017; Tay & Wonkyi, 2018; Zhang et al., 2023; Zulnaidi & Zamri, 2020).

Table 15. The utilization of GeoGebra software in different countries

Author/year	Country	Title	Topic	Finding	Remaining Issues	Grade
(Herceg & Herceg, 2010)	Serbia	Numerical integration with GeoGebra in high school	Numerical Integration	GeoGebra can calculate the value of the Riemann sum of some other approximate formula and can also produce appropriate graphics	GeoGebra tools enable the students to independently identify each step of numerical integration and visually represent these steps.	High school
(Zulnaidi et al., 2020)	Indonesia	Effect of use of GeoGebra on achievement of high school mathematics students	Function and limit function	GeoGebra can illustrate mathematical concepts and procedures well through visuals and graphs of functions and limit functions	Intervention for increasing student achievement	11
(Martinez, 2017)	Central California, US	The effect of using GeoGebra	Right triangle and trigonometry	Using GeoGebra helps	We need to continue to assess the	12

		on students' achievement		students improve and engage in high-level mathematical concepts	effectiveness of using iPads during instruction	
(Tay & Wonkyi, 2018)	Central Region of Ghana	The effect of using GeoGebra on senior high school students' performance in circle - theorems	Circle theorem	Students who use GeoGebra understand the circle theorem better than those who do not use it with the circle theorem	GeoGebra should consider the use of Plane Geometry I, such as the exterior angle theorem and properties of special triangles (isosceles and equilateral)	11
(Em & Roman, 2020)	Philippines	Effectiveness of GeoGebra in teaching grade 10 mathematics	Cartesian plane, distance formula, midpoint formula, equation of a circle, polynomial function	- GeoGebra is effective in teaching and learning the Cartesian plane, midpoint formula, equation of a circle, and polynomial function but has no significant effect on the distance formula - GeoGebra helps students	Integrate GeoGebra in teaching and learning distance formulas and utilize teachers' training in GeoGebra in teaching and learning mathematics in ASEAN region	10

				grasp problem-based and research-based mathematics learning.		
(Zhang et al., 2023)	University of Hong Kong, China	Dynamic Visualization by GeoGebra for mathematics learning: a meta-analysis of 20 years of research	Calculus and geometry	Nineteen effect sizes were synthesized from fourteen studies in the last two decades, with 1,334 participants. The analysis results demonstrated a positive medium – to – large effect (Hedges's $g=0.653$ ) of GeoGebra as a dynamic visualization tool for improving students' mathematics achievement.	Integrating GeoGebra into the topic of calculus geometry and some pedagogical implication with the learners less than 50.	High school

Source: Developed by the author based on a literature review

Many countries, such as Serbia, Indonesia, the US, and the Philippines, use GeoGebra software to support teaching and learning mathematics in upper secondary education. Implementing GeoGebra software results in positive improvements in many areas, such as numerical integration, functions, limit functions, circle theorems, right triangles, trigonometry, Cartesian formulas, distance formulas, the equation of a circle,

polynomial functions, calculus, and geometry (Em & Roman, 2020; Herceg & Herceg, 2010; Martinez, 2017; Tay & Wonkyi, 2018; Zhang et al., 2023; Zulnaidi & Zamri, 2020).

Moreover, GeoGebra software also provides the following benefits in supporting students and teachers:

- It has many features that are compatible with providing VR and CU in learning mathematics, particularly geometry (Jelatu & Ardana, 2018; Zulnaidi et al., 2020).
- It is user-friendly and promotes interaction between teachers and students (Ziatdinov & Valles Jr, 2022; Zulnaidi et al., 2020)
- It is a powerful software package that can influence students' mathematics achievement and reduce misconceptions (Birgin & Topuz, 2021; Zulnaidi et al., 2020).
- It is open-source and available both online and offline (Albano & Dello Iacono, 2019; Zhang et al., 2023)

On the other hand, a vector is a geometrical entity represented by a directed line segment. It is characterized by magnitude (length), which represents the size or quantity of the vector, and direction, which represents the vector's orientation in a plane and space (Anton & Rorres, 2013). Vectors form the foundation of geometry and are related to many high-level mathematical concepts, bringing geometry and algebra together in relation to direction, movement, and magnitude, including visualization skills for upper secondary education. Students need to grasp vector concepts to understand more complex concepts in their studies and connect with other subjects (Dray & Manogue, 2023; Wrede, 2013). In plane and space geometry, many scholars have explored vector spaces, often focusing on what is termed "high visualization" (Tay & Wonkyi, 2018; Zhang et al., 2023).

According to the national examination report for the academic year 2021-2022, students have low achievement in geometry, particularly in vector content (MoEYS, 2021). Vector content consists of geometric shapes and algebra. It was included in the geometry domain of the Cambodian mathematics curriculum for general and technical education, and the textbooks for upper secondary education in grade 10, which are implemented in both the science and social science tracks (MoEYS, 2018f, 2018g).

Students face challenges in understanding and applying vector concepts, such as grasping their meaning, understanding the relationship among their representations and operations, navigating different representations, and applying knowledge flexibly to explain, represent, and solve abstract or contextual problems involving vectors (Latifa et al., 2021; Sabah, 2023). Therefore, in relation to these previous studies, vector contents are selected to develop the intervention in this study, which measures the effectiveness of integrating GeoGebra software into teaching and learning geometry in upper secondary schools in Cambodia.

## 2.4 SUMMARY AND CONCEPTUAL FRAMEWORK

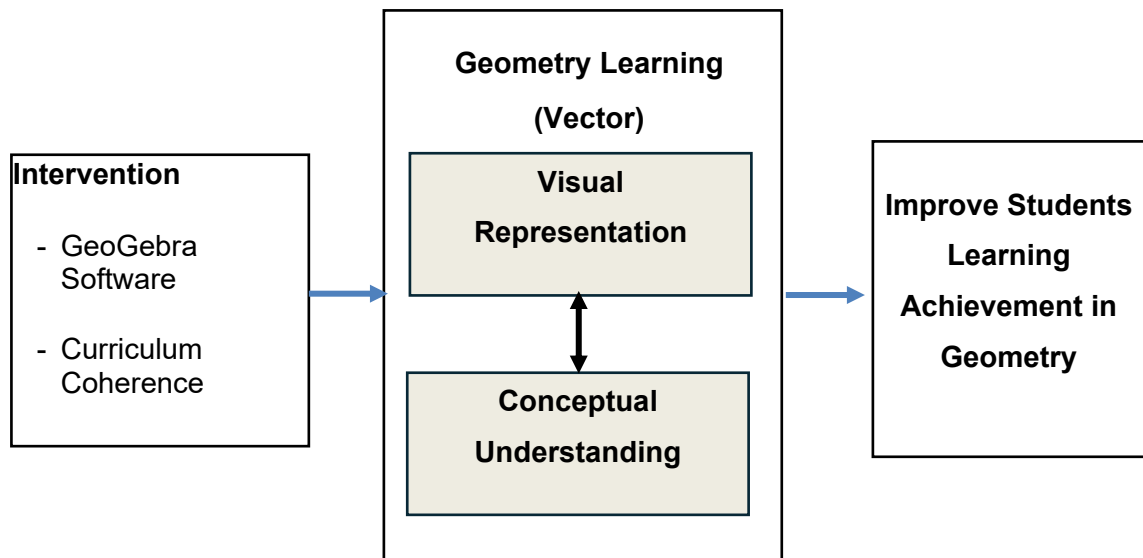
Section 2.1 provides an overview of geometry, including its historical background and the types of geometry commonly used in general education. It highlights the importance of geometry in a global and Cambodian context. Additionally, it presents how geometry is implemented in Cambodian education and how the geometry curriculum has been introduced, particularly at the upper secondary education level. It also discusses curriculum coherence, which various scholars identify as a key predictor of overcoming students' low achievement. It establishes the operational definition for analyzing the Cambodian curriculum from the perspective of curriculum coherence, focusing on content relations in the geometry domain. This research defines ***curriculum coherence as the relationship among corresponding domains, sub-domains, contents, sub-contents, and learning outcomes within and across grades, all organized in order as a whole***. The last part of this section also discusses CU. ***CU is defined as the students' ability: (1) to explain the mathematical concept following the mathematical procedure, including calculation (Properties); (2) to provide the justifications for the mathematical procedure (Reason); and (3) to use the mathematical expression to give the reason in relation to the VR (Representation)***.

Section 2.2 focuses on the discussion of visualization and its relationship with CU. The literature review reveals that visualization consists of three main aspects: visual perception, visual-spatial skills, and VR. These three aspects are implemented at different levels of general education. In primary education, the focus is on visual perception; this leads to visual-spatial skills in lower secondary education, and finally, the emphasis shifts to VR in upper secondary education. It also describes the relationship among the three aspects. Moreover, while this study mainly targeted the

upper secondary school, it describes the relationship between CU and VR in geometry. Therefore, understanding how visualization works together in geometry can lead to more effective teaching techniques, better learning success, and a deeper understanding of the subject, which can help students develop a CU of geometry. While this study mainly focuses on upper secondary education, the VR is focused. In this study, ***VR is defined as the student's ability: (1) to identify and interpret visual information, including visual, symbolic, and verbal representations (Recognition); (2) to transform and coordinate different types of picture representations in mind (Mental Operation); and (3) to create geometrical drawings to clarify the problem (External Representation).*** The relationship between visualization and CU has been clarified.

Section 2.3 introduces some approaches to overcoming student learning difficulties in geometry, including DGS and curriculum coherence. The relationship between DGS and CU and DGS and curriculum coherence has been discussed. Moreover, it described DGS, particularly GeoGebra software, as one possible approach to overcoming students' difficulties in learning geometry, including VR and CU. It also describes GeoGebra software, its features, and its strengths. Moreover, this section presents the global trend of integrating GeoGebra software into upper secondary education, which various scholars have mentioned. At the end of this section, the critical contents in the geometry domain have been identified for developing the intervention lessons, specifically vector contents.

Through the previous literature review from various scholars and the national examination report, vector contents have been selected as the critical topic to develop the lesson intervention to measure the effectiveness of integrating GeoGebra into teaching and learning geometry at upper secondary schools in Cambodia. Therefore, the conceptual framework has been developed in this study, as shown in the figure below.



*Figure 9. The conceptual framework of this study*

Source: The author developed this based on the literature review.

This conceptual framework shows how integrating GeoGebra software and ensuring curriculum coherence can help students learn vectors in geometry more effectively. These interventions are designed to support two key aspects of learning: VR and CU, following the above definition. These two aspects support each other. By improving both, students' learning achievement in geometry will increase.

## **CHAPTER 3: RESEARCH METHODOLOGY**

This chapter explains two main procedures: (1) curriculum analysis methods in responding to research question 3, and (2) the process of the field survey for research question 4. Thereafter, it presents the research ethics followed in this study.

### **3.1 METHODOLOGY OF GEOMETRY CURRICULUM ANALYSIS**

#### **3.1.1 Data Source**

This study analyzed four recently revised mathematics syllabi published in 2018 (MoEYS, 2018d, 2018e, 2018f, 2018g) and two Singaporean mathematics syllabi published in 2020–2021 (MOE, 2020, 2021). These were the latest versions available during the research period of 2023–2024. They cover primary education (grades 1–6), lower secondary education (grades 7–9), and upper secondary education (grades 10–12) in general education. The syllabi for grades 10–12 are separated into two files: one for the science track and the other for the social science track. All syllabi contain goals, objectives, domains, contents, sub-contents, and learning outcomes from grades 1 to 12. In this research, the Cambodian curriculum is comparatively analyzed with the Singaporean curriculum. This is because Singapore is an economic leader among the ASEAN countries, which share backgrounds and historical contexts, and its curriculum is developed with coherence, which is indicated in the curriculum framework and syllabi. The Singaporean mathematics syllabus for primary education was published in 2021, and the mathematics syllabus for secondary education was published in 2023. The selection of the Singaporean curriculum is based on some reasons: (1) According to Trends in International Mathematics and Science Study (TIMSS), the Singaporean mathematics curriculum is highly appreciated for several reasons by different scholars and has resulted in Singaporean students consistently ranking at the top in international mathematics assessments such as TIMSS and Programme for International Student Assessment (PISA) (Kadijevich et al., 2023; NIEI & PRI, 2021). (2) The Singaporean mathematics curriculum is often noted for its coherence (Grey, 2020; Hairon, 2021; MOE, 2020). (3) The curriculum review report on the Cambodian Mathematics Curriculum in 2021

highlighted that the Singaporean mathematics curriculum was developed with a focus on coherence (NIEI & PRI, 2021).

### **3.1.2 Data Analysis Method**

All the Cambodian mathematics syllabi were translated from Khmer into English. They were extracted and reorganized. Then, the geometry domain was categorized into five sub-domains: plane geometry, solid geometry, relations and transformations, constructions, and vectors. Contents and sub-contents were translated directly from the original text, and those within geometry were arranged according to domains and sub-domains. Similarly, the learning outcomes were translated and summarized under knowledge, skills, and attitudes. The translation aims to facilitate consultations with mathematics experts regarding the validity and reliability of the analysis. The original English versions of the Singaporean mathematics syllabi were used.

Then, the data were input into an Excel sheet. The domains, sub-domains, contents, sub-contents, and learning outcomes were arranged from grades 1 to 12 and examined within the same grade and across grades to check whether they corresponded to each other.

This study analyzed the domains, sub-domains, contents, sub-contents, and learning outcomes using general topics trace mapping (GTTM). GTTM was developed for content analysis based on the Trends in International Mathematics and Science Study (TIMSS) (Schmidt et al., 2005a). It provides a way to compare and analyze curricula across countries, identify the content at each grade level, and linkages between contents, sub-contents, and learning outcomes. Based on the GTTM, the following procedure was set:

- (1) Collect all the geometry contents from the syllabi and arrange them into appropriate sub-domains per grade. Then, examine the contents within the sub-domains to determine whether they are repeated or sequenced.
- (2) Collect sub-contents from the syllabi and arrange them under each content in order, within and across grades. This step identifies the relation between contents and sub-contents within each grade and across grades to indicate how they fit together.

- (3) Analyze the relation between sub-contents and learning outcomes within the sub-content of each grade level. Then, examine how each learning outcome fits together with the sub-contents within each content at the grade level.
- (4) Conduct an overall analysis of the contents, sub-contents, and learning outcomes across domains, sub-domains, and grades. This step shows the result of curriculum coherence as shown by the relation between corresponding domains, sub-domains, contents, sub-contents, and learning outcomes within and across grades.

The Singaporean curriculum was examined in the same way. Then, the following procedure was set for comparative analysis with the Singaporean curriculum:

- (1) The constructed curriculum structures of Cambodia and Singapore are compared in terms of aims and contents.
- (2) The sub-domains and contents are compared in terms of organization and age-appropriateness within and across the grades.
- (3) The overall analysis of the contents, sub-contents, and learning outcomes of both countries are compared in terms of correspondence.

## 3.2 METHODOLOGY OF FIELD SURVEY

### 3.2.1 Research Location and Participants

The research was conducted in Cambodia. Two upper secondary schools located in Phnom Penh, the capital city of Cambodia, were selected for the field survey. Figure 10 shows the research location.

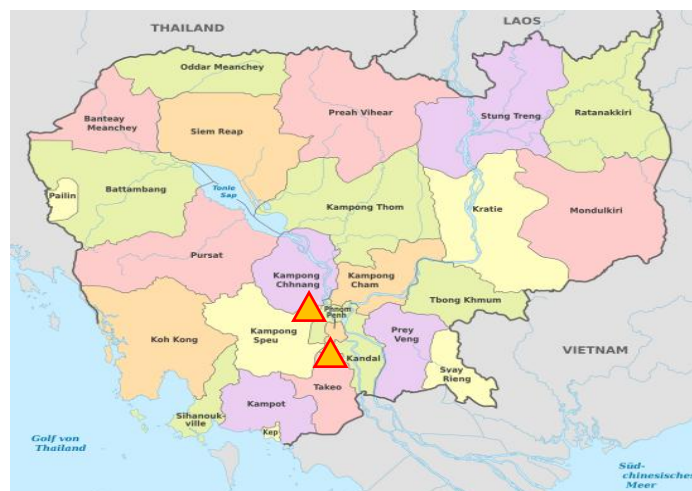


Figure 10. A map of the Kingdom of Cambodia showing the research location.

For convenience, the areas for the field and pilot surveys were chosen from different schools. The pilot survey was conducted in Kondal province, while the field survey was conducted in Phnom Penh city, Kingdom of Cambodia. The reason for choosing different schools is to prevent students from remembering the test items for the field survey. Meanwhile, two upper secondary schools were selected to implement the field surveys.

The participants of this study included upper secondary school students in grade 10 from two schools (School A and School B) in Phnom Penh, the capital city of Cambodia, during the 2022 – 2023 academic year. Both schools have similar characteristics, including those related to location, economics, and infrastructure. Two classes were chosen from each school, one assigned as the experimental group and the other as the control group. The experimental group received GeoGebra software instruction, while the control group used the traditional teaching method. After data were cleaned, in total, 168 (103 females) high school students participated in both tests, the pre-test and post-test, of which 100 were in the experimental group (75 (50 females) from school A) and 25 (12 females) from school B) and 68 in the control group (40 (23 females) from school A) and 28 (18 females) from school B). It is important to note that the selected participants were based on the students in the list provided by the target schools. Among them, 12 students were selected for interviews to supplement their responses and assess their understanding of VR and CU. Additionally, three curriculum developers from MoEYS who responsible for the Department of Curriculum Development (DCD), Cambodia, were involved in the curriculum development process, and two teachers teaching in grade 10 with the same qualification (Master's degree) and same duration of their teaching experiences (seven years) at the target school were selected for interviews to triangulate the findings. Table 16 shows the summary of the participants.

Table 16. Summary of the participants

Categories	Experiment	control	Total	Female
Curriculum Developers			3	0
Teachers			2	0
Grade 10 students from school A	75 (50)	40 (23)	115	73
Grade 10 students from school B	25 (12)	28 (18)	53	30
	<b>100 (62)</b>	<b>68 (41)</b>		

### 3.2.2 Research Design and Procedure

The study employed a quasi-experimental, non-equivalent group research design with a control and experimental group following by pretests and post-tests. This design ensures that the researcher does not assign participants randomly, meaning that the participants were selected by the school's design. It also allows the researcher to recognize a control group comparable to the experimental group in terms of pre-intervention characteristics (Cook et al., 1979; Creswell & Creswell, 2018; Gay et al., 2012). The study was designed to examine the impact of using GeoGebra software on students' VR and CU of geometry, specifically focusing on vectors and operations on vectors. There are several reasons for selecting vector contents:

- (1) Vectors are the foundation topic addressed in Cambodia's first grade of the upper secondary school mathematics curriculum (MoEYS, 2018h, 2018i). It is basic knowledge for upper secondary school students in Cambodia context. Students' understanding of the vector concept can help them understand other related topics more easily.
- (2) A vector is the basic concept of geometry, which is related to many advanced topics in mathematics, such as computer graphics, linear algebra, physics, engineering, and navigation. Students need to understand vectors to grasp more complex concepts in their studies.
- (3) The results of the curriculum analysis revealed that the vector content lacked coherence within and across grade levels.
- (4) Vectors were one of the topics indicating students have poor achievement in the national examination in grade 12 in the 2021–2022 year; 47.93% answered correctly.

The intervention was adapted to the current Cambodian textbook for grade 10, published in 2020, with the contents of vectors and operations on vectors from pages 106 to 133, including exercises. This content is also addressed in the current mathematics curriculum in grade 10, published in 2018. Table 17 shows the summary of the intervention design for both groups.

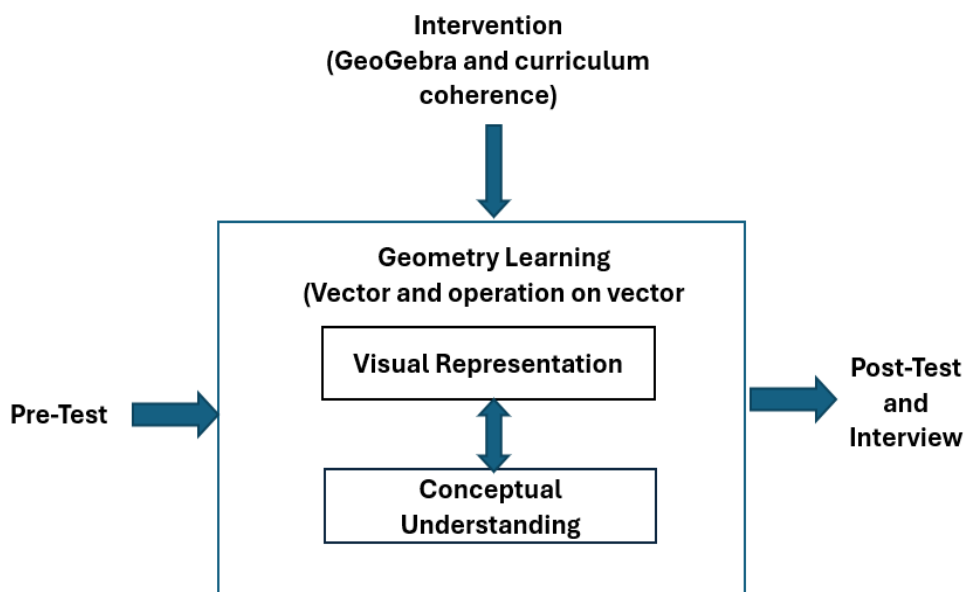
*Table 17. Summary of the intervention design for control and experimental groups*

<b>Group</b>	<b>Lesson</b>	<b>Test</b>	<b>Intervention</b>
Experiment Group	Chapter 1. Vectors and operations on vectors 1.1 Definition of vectors 1.2 Addition and subtraction of vectors 1.3 Multiplication of vectors with a real number 1.4 Coordinates of a vector in a plane 1.5 Calculation of the coordinate of the vector 1.6 Scalar product	Pre-test and post-test design with a focus on VR and CU	. Integrating GeoGebra software . Geometry curriculum
Control Group	Chapter 1. Vectors and operations on vectors 1.1 Definition of vectors 1.2 Addition and subtraction of vectors 1.3 Multiplication of vectors with a real number 1.4 Coordinate of a vector in a plane	Pre-test and post-test design with a focus on VR and CU	. Traditional teaching methods . Geometry curriculum

- 
- 1.5 Calculation of the coordinate of the vector
  - 1.6 Scalar product
- 

The implementation procedure started with the pre-test, followed by the intervention, and finally, the post-test and supplementary student interviews. There were six sub-contents under vector and operation on vector content: definition of vector, addition and subtraction of vectors, multiplication of vector by real numbers, coordinate of a vector in a plane, calculation of the coordinate of the vector, and scalar product. However, three sub-contents were selected to organize the pre-test and post-test, while the other three were embedded.

The intervention lesson was done following the research framework. Figure 11 shows the framework and procedure of the intervention lesson.



*Figure 11. The research framework and procedure of the intervention lesson*

Source: The author developed based on the literature review.

The procedure used quasi-experimental sampling with approval from the school principals, emphasising ethical considerations and cooperation. All available students from the selected schools were invited to participate in completing the test. It is important to note that the school principals determined the selection process and class assignments. The participants of both groups were not equal in number but

rather reflected the actual class setting. Some participants were removed from the final analysis because they did not participate fully in the process, such as not participating in either the pre-test or post-test.

### **3.2.3 Research Instrument**

The researcher developed two sample lesson plans before the teaching practice period to guide teachers at School A and School B on how to integrate GeoGebra into geometry content (**see Appendices 2 and 3**). The lesson plans were developed with the same content, followed by sub-contents and the format of Cambodian lesson plans for the experimental and control groups. The difference between the two groups is only the instruction method. The experimental group used GeoGebra software for integrating teaching and learning geometry, while the control group used traditional teaching methods without using GeoGebra software. It is important to note that there are five hours per week of actual class teaching for mathematics. Only two hours were allocated for teaching geometry content designed by both school teachers. Each lesson plan was developed for classroom teaching within two hours.

In the whole field survey process, teachers developed four progressive lesson plans (**see Appendix 4**), following the sample lesson plans provided by each control and experimental group. The lesson plans were developed following sub-contents based on the weekly schedule of teacher instruction. The differences between the development of lesson plans for both groups include the use of GeoGebra software for shape drawing in the experimental groups, while the control groups employ hand drawing. Additionally, the teachers' instructional strategies differ, with the experimental groups being instructed through the integration of GeoGebra software, whereas the control groups utilize traditional teaching methods. The experimental group integrated GeoGebra software into the lesson with vectors and operations on vectors, and the control group was taught the same lesson using traditional methods. The instructions were given following the implementation period for the field survey. The teaching practice was implemented for five weeks, from June 12, 2023, to July 22, 2023. The classroom intervention process started with pre-tests and ended with post-tests and interviews. Table 18 shows the process of all the implementation activities.

Table 18. The implementation period for the field survey

Weeks	contents	Tools	Textbook Page
1	<ul style="list-style-type: none"> <li>- Pre-tests were conducted in the two schools</li> <li>- Two teachers were guided using a sample lesson plan</li> <li>- Sample Chapter: Vector plane and Vector space               <ul style="list-style-type: none"> <li>1.1 Coordinate of a vector</li> <li>1.2 Length of vector</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Pre-test</li> <li>- Sample lesson plan</li> <li>- Lesson plan</li> <li>- Intervention</li> <li>- Classroom observation and videos recorded</li> </ul>	- Teacher-guided pages 325-368, published in the year 2007.
2	Chapter 1. Vectors and operations on vectors <ul style="list-style-type: none"> <li>1.1. Meaning of vectors               <ul style="list-style-type: none"> <li>a. Meaning of vector</li> <li>b. Equality vectors</li> <li>c. Opposite vectors</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Lesson plan</li> <li>- Worksheets</li> <li>- Intervention</li> <li>- Classroom observation and videos recorded</li> </ul>	- 106 - 108
3	Chapter 1. Vectors and operations on vectors (continuous) <ul style="list-style-type: none"> <li>1.2. Addition and subtraction of vectors               <ul style="list-style-type: none"> <li>a. Addition of vector (continuous)</li> <li>b. Subtraction of vectors</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Lesson plan</li> <li>- Worksheets</li> <li>- Intervention</li> <li>- Classroom observation and videos recorded</li> </ul>	- 108 - 111
4	Chapter 1. Vectors and operations on vectors (continuous) <ul style="list-style-type: none"> <li>1.3 Multiplication of vectors with real numbers               <ul style="list-style-type: none"> <li>. Vector Unit                   <ul style="list-style-type: none"> <li>a. Coordinate of a vector in a plane</li> </ul> </li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>- Lesson plan</li> <li>- Worksheets</li> <li>- Intervention</li> <li>- Classroom observation and videos recorded</li> </ul>	- 112 - 119

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b. Coordinate vectors

5	Chapter 1. Vectors and operations on vectors (continuous)	- Guidance and feedback, develop worksheets	- 119-133
	1.4 Calculate the coordinate of the vector	- Classroom observation, videos recorded	
	1.5 Scalar Product of a Vector	- Post-tests were conducted in the two schools	
		- Interview with 12 students who achieved a low, medium, and high score based on the pre-test answers.	

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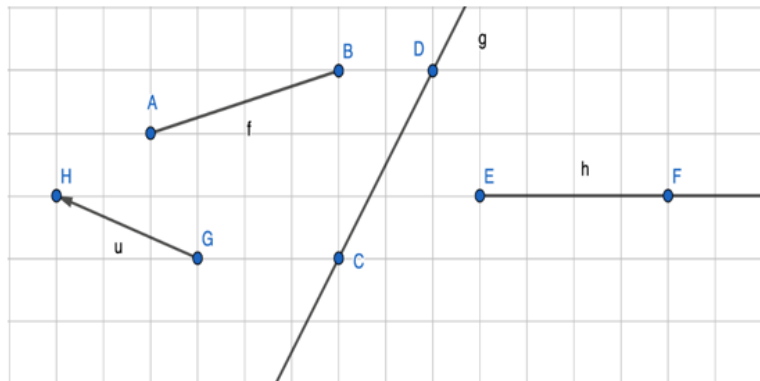
Source: The author developed based on the Cambodia textbook in grade 10.

Moreover, thirteen worksheets were used in the implementation practices to monitor the students' activities and assess their learning progress during the teachers' instruction. These worksheets were used for student practice in both groups to see how students calculate and their interaction (**See Appendix 5**). The figure below shows a sample of the worksheet for assessing students' learning activities.

*Worksheet*

1. A segment that has a direction from A to B in the plane is called  
A. Line AB      B. Segment AB      C. Vector AB      D. Other answer

2. In the figure below, which one is a vector ?



3. How many equal vectors are in the figure below ? .....



*Figure 12. A sample worksheet*

The above worksheet is used for the first week of the implementation. It included three tasks to measure the students' CU and VR of the sub-content of the “meaning of vectors”. The purpose of this task is to see how students are able to explain the calculation procedure and differentiate the concept of vector, vector equality, and the opposite of a vector through the provided shapes.

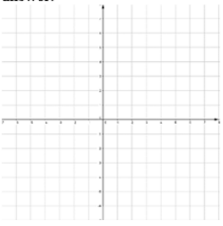
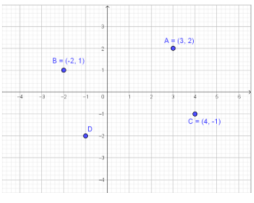
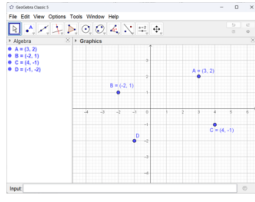
Moreover, all tasks were developed to assess the students' CU and VR.

The same pre- and post-tests were administered to the control group, and the same tests were used for the experimental group. The purpose of the tests is to evaluate students' depth of CU and their ability to represent vector-related concepts visually. This involves determining how well students grasp the underlying principles of vectors, such as magnitude, direction, and component form, and how effectively they can translate these abstract concepts into accurate VR, such as shapes or coordinate-based drawings. By analyzing both their CU and VR, the assessment provides insight into students' integrated mathematical thinking, helping teachers

identify specific strengths and learning gaps in their understanding of vector concepts in geometry.

The test items were adapted from the contents of the Cambodian curriculum and the textbook for grade 10, which was published in 2020 (MoEYS, 2018f). The Cambodian Mathematics Curriculum for upper secondary school was published in 2018 (MoEYS, 2018f, 2018g). The table below shows an example of how to develop pre-tests, post-tests, and an intervention using GeoGebra connected with VR and CU through the curriculum coherence.

Table 19. Sample of pre-test and post-test for the field survey

Test Item	Conceptual Understanding	Visual Representation	Curriculum and textbook	Intervention
<p>1. Set four points which are given A(3,2), B(-2,1), C(4,-1) and D(-1,-2) in the below figure and explain how you get the answer.</p> 	<p>The students' abilities to:</p> <ul style="list-style-type: none"> <li>explain the mathematical concept following the mathematical procedure, including calculation (<b>Properties</b>)</li> <li>give the reason for the mathematical procedure (<b>Reason</b>)</li> <li>use the mathematical expression to give the reason in relation to the visual representation (<b>Representation</b>).</li> </ul> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>By the given problem, we can set the points as below:</p> <ul style="list-style-type: none"> <li>- Point A(3,2) at the first quadrant because <math>x_A = 3 &gt; 0</math> and <math>y_A = 2 &gt; 0</math></li> <li>- Point B(-2,1) at the second quadrant because <math>x_B = -2 &lt; 0</math> and <math>y_B = 1 &gt; 0</math></li> <li>- Point C(4,-1) at the third quadrant because <math>x_C = 4 &gt; 0</math> and <math>y_C = -1 &lt; 0</math></li> <li>- Point D(-1,-2) at the fourth quadrant because <math>x_D = -1 &lt; 0</math> and <math>y_D = -2 &lt; 0</math></li> </ul> <p><b>Figure:</b></p> <ul style="list-style-type: none"> <li>- Set the four points at the right position</li> </ul> </div>	<p>The students' abilities to</p> <ul style="list-style-type: none"> <li>recognise and interpret visual information, including visual, symbolic, and verbal representations (<b>Recognition</b>)</li> <li>transform and coordinate different types of picture representations in the mind (<b>Mental Operation</b>)</li> <li>create geometrical drawings to clarify the problem (<b>External Representation</b>).</li> </ul> 	<p>Mathematics Curriculum and textbook of 10<sup>th</sup> grade.</p>	<ul style="list-style-type: none"> <li>GeoGebra can help students explore and clarify the result by showing mathematical expressions in response to CU and VR.</li> <li>Students are confident after seeing the proof from the GeoGebra interface, which shows algebraic and geometric views on the screen in response to CU and VR.</li> </ul> 

The purpose of test item 1 is to assess students' ability to solve geometry problems through VR and CU. In terms of conceptual understanding, students are required to explain the reasoning behind the placement of points on the Cartesian coordinate plane. This involves identifying the properties of coordinate pairs, specifically understanding how the signs of the  $x$ - and  $y$ -coordinates determine the location of each point in one of the four quadrants. Students must follow a mathematical procedure to justify their answers, such as stating that point A(3,2) lies in the first quadrant because both  $x$  and  $y$  are positive. This aligns with the components of CU: applying properties, providing reasons, and using mathematical expressions to represent and explain their reasoning. At the same time, the item requires students to engage with VR in several ways. First, they must recognize and interpret visual information by reading and understanding the layout of the Cartesian plane. Second,

they are expected to mentally transform symbolic data (coordinate pairs) into spatial positions involving mental operations. Third, students must externally represent the solution by plotting the given points accurately on the graph. These actions correspond to the core elements of VR: recognition, mental operation, and external representation.

The task encourages the integration of CU and VR by requiring students to justify their reasoning (CU) while verifying and communicating their understanding through graphing (VR). This dual engagement promotes deeper learning as students explain and visualize their solutions. Furthermore, the GeoGebra software intervention supports this integration by enabling students to explore and confirm their answers through dynamic algebraic and geometric representations, reinforcing both their CU and VR skills. Following the above procedure, the other four test items were developed.

The test items consist of five open-ended questions extracted from three sub-contents: (1) “meaning of a vector,” which assesses the student's understanding of the foundation of the vector concept, including the definitions of a vector and type of a vector; (2) “addition and subtraction of vectors,” which assesses the student’s knowledge on the calculation procedure and their critical thinking demonstrated of a solving geometry problems with hidden information; and (3) “coordinate of a vector in a plane,” which focuses on the student’s generalization of vector concepts and their initial ideas on how to solve a problem with abstract information. Moreover, the “scalar product of vectors” was embedded in the “coordinate of a vector in a plane” content, while the students used the “multiplication of vectors with real numbers” concept through the moving coordinate of points. Each test item consisted of both CU and VR concepts. Even though the other sub-contents in “Chapter 1. Vector and operation on vector” are not selected, they are embedded in these three. Table 20 shows the development of the test items.

Table 20. Test items development

Sub-content	Test items of VR	Test items of CU	Curriculum and textbook
Meaning of a vector	1, 2	1, 2	
Addition and subtraction of vectors	3, 4	3, 4	Cambodian mathematics curriculum and textbook of grade 10
Coordinate of a vector in a plane	5	5	
<b>Total number of test items</b>	<b>5</b>	<b>5</b>	

Source: The author developed the table by adapting from the Cambodian mathematics curriculum and textbook of grade 10 (MoEYS, 2020c).

Table 20 shows that three sub-contents covered test items. Each test item was constructed to measure students' understanding of VR, focusing on students drawing the shapes, and CU, focusing on students' explanation of the process of drawing geometric shapes and their calculation procedure in solving geometry problems. Test items 1 and 2 were covered by the sub-contents of the "meaning of a vector". This test intends to assess the student's abilities to identify, locate, and justify the placement points on a coordinate plane, integrating both their CU of the quadrant systems and their VR in coordinate geometry through test item 1, then measure the ability of the students to construct and interpret vectors between points on a coordinate plane and analyze their properties through visual and conceptual reasoning. By assessing these test items 1 and 2, we identified the students' understanding of VR and CU with a focus on the concept of "definition of vectors."

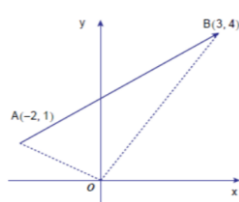
The test items 3 and 4 were covered by the sub-content of "addition and subtraction of vectors." These test items are designed to assess students' abilities to interpret hidden information, apply vector components to an initial point, and perform vector operations accurately. These test items evaluate students' ability to interpret a

given vector visually on a coordinate plane and express it accurately in component form by demonstrating their CU of vector direction and magnitude and their VR skills in coordinates.

Test item 5 was dealt with by the sub-content of “coordinate of a vector in a plane.” This test is intended to assess students' critical thinking skills by requiring them to apply prior knowledge of vector operations, embedding multiplication of vectors with real numbers and scalar products of vectors. It evaluates their ability to integrate and use their understanding to solve the problem effectively by applying it to the component of a vector in a plane.

Even though Cambodian mathematics textbooks do not directly express CU and VR, they were already embedded in visualizing and explaining. In addition, in the textbook, to make it easier for the students to understand the geometry lesson, conceptual understanding is necessary, and to show the process of calculation, a visual representation is required. The table below describes how VR and CU are embedded in the textbook in grade 10.

Table 21. The sample of VR and CU expressed in textbook grade 10

Translation	Explanation (CU)	Geometric shape (VR)
<p><b>Sample exercise:</b> Given points A(-2,1) and B(3,4). a. Calculate the coordinates of vectors <math>\overrightarrow{AB}</math>. b. Calculate the length of <math> \overrightarrow{AB} </math></p> <p><b>Solution:</b> a. Calculate the coordinates of vectors <math>\overrightarrow{AB}</math>. Through the picture, we have <math display="block">\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}</math><math display="block">= (3\vec{i} + 4\vec{j}) - (-2\vec{i} + \vec{j})</math><math display="block">= (3\vec{i} + 2\vec{i}) + (4\vec{j} - \vec{j}) = 5\vec{i} + 3\vec{j}</math>Therefore, the coordinates of <math>\overrightarrow{AB} = (5, 3)</math>. b. Calculate the length of <math> \overrightarrow{AB} </math> <math display="block"> \overrightarrow{AB}  = \sqrt{5^2 + 3^2} = \sqrt{34}</math></p>	<p><b>លំហាត់គំនូរ</b> គេមានចំណុច A(-2, 1) និង B(3, 4) ។ ក. គណនាកូអរដោនេនៃ <math>\overrightarrow{AB}</math> ខ. គណនាប្រវែង <math> \overrightarrow{AB} </math> ។</p> <p><b>ដំណោះស្រាយ</b> ក. គណនាកូអរដោនេនៃ <math>\overrightarrow{AB}</math> តាមរូប គេបាន <math display="block">\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}</math><math display="block">= (3\vec{i} + 4\vec{j}) - (-2\vec{i} + \vec{j})</math><math display="block">= (3\vec{i} + 2\vec{i}) + (4\vec{j} - \vec{j}) = 5\vec{i} + 3\vec{j}</math>ដូចនេះ កូអរដោនេនៃ <math>\overrightarrow{AB} = (5, 3)</math> ។ ខ. គណនាប្រវែង <math> \overrightarrow{AB} </math> <math display="block"> \overrightarrow{AB}  = \sqrt{5^2 + 3^2} = \sqrt{34}</math> ។</p>	

This table demonstrates how CU and VR work together to help students learn about vectors. It shows how a vector  $\overrightarrow{AB}$  is calculated from two points using coordinate

subtraction, and its magnitude is found using the Pythagorean Theorem. The graph visually confirms the vector's direction and length, reinforcing the connection between algebraic reasoning and spatial visualization.

The table shows that the vector task integrates both CU and VR. For CU, students apply the concept of vector subtraction by calculating  $\vec{AB} = \vec{OB} - \vec{OA}$ , determine the components as  $5\vec{i} + 3\vec{j}$ , and compute the magnitude  $|\vec{AB}| = \sqrt{34}$ , demonstrating the student's understanding of vector properties, procedures, and reasoning. For VR, the geometric shape helps students recognize the positions of  $A$  and  $B$  on the coordinate plane, mentally visualize the direction and length of the vector, and externally represent  $\vec{AB}$  with an arrow. This supports recognition, mental operation, and external representation.

### 3.2.4 Validation of the Instrument

As mentioned above, the test items were piloted twice to ensure construct validity, that the students understood them, and that the provided hours for solving the problems were appropriate. Based on the respondents' results, the common responses in the pilot test were identified, improving the main pre-test and post-test development. The pilot test items were developed as detailed in **Appendix 6**.

Three experts validated the test items: two were teachers familiar with vector concepts from their experiences teaching grade 10, and another expert was a teacher from the National Institute of Education (NIE) of Cambodia, majoring in mathematics. These experts reviewed the test items to ensure they were clear, relevant, and aligned with the curriculum and textbook and met the current context. The reliability of the first pilot test items, measured by Cronbach's Alpha, was 0.786; the allotted one hour (fifty minutes in a teaching hour) was insufficient for the students to complete all the problems. As a result, test item 1 was split into two because it originally contained two distinct questions. After confirmation from the expert and teachers, test item 5 was deleted because its content was beyond the scope of grade 10 students.

The second pilot was conducted to confirm if the revised time limit was acceptable. It confirmed that the test items could be measured with a Cronbach's Alpha of 0.904, indicating that they are reliable. Moreover, the duration of 40 minutes was suitable for completing all test items, suggesting that they can be used. The final development test items were used in pre-tests and post-tests for the actual

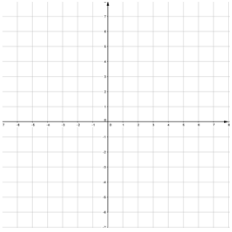
implementation during the intervention period. Figure 13 shows the results of the developed test items.

Name of School: .....  
 Name of Student: ..... Sex: .....  
 Grade: ..... Age: .....

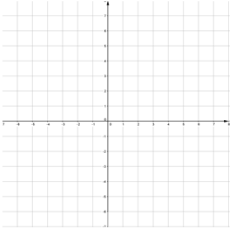
**Instruction:**  
 Please write all your answers on this paper only. The answer must be solved by drawing the figure in your answer sheet for each solution. The test takes the time only 30 minutes.

**Problem**

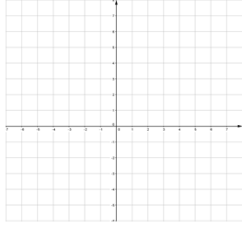
- Set four points which are given A(3,2), B(-2,1), C(4,-1) and D(-1,-2) on the below figure and explain how you get the answer.



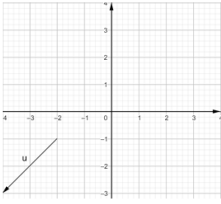
- Following exercise number 1, draw the vector  $\vec{AB}$  and  $\vec{DC}$ . What can you see on both vectors? Please explain by drawing the below figure.



- Draw the vector with an initial point (2,3), then find the endpoint by giving a vector in component form  $\vec{v} = (2, 4)$  and explain, how you get such an answer by showing the graph.



- Write a vector  $\vec{w}$  in component form to represent the below shape and explain your answer.



- Graph  $\vec{p} + \vec{q}$  with the same initial point as  $\vec{p}$  and  $\vec{q}$ . Explain how you get the answer.

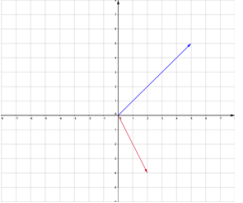


Figure 13. Pre-test and post-test items of the field survey.

### 3.2.5 Data Collection

The data collection was conducted using the same procedure at both schools. It was collected through tests and interviews administered before and after the intervention. Three steps were followed.

First, the pre-test with five test items was conducted for the experimental and control groups before the intervention lessons. The test items were distributed to all students. Once completed, they were collected to be marked by the two teachers and verified by the expert. The tests were marked carefully using a scoring rubric. Both groups for each school followed this process.


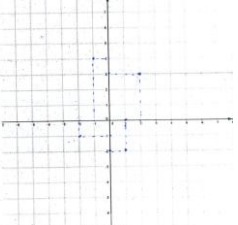
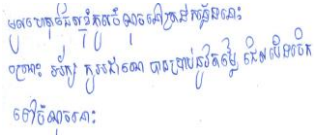
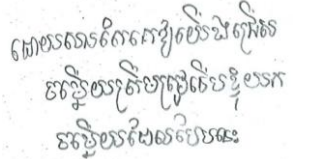
Second, 13 worksheets were collected and administered during the intervention period to investigate student calculation procedures during the intervention in the classroom. This provided a much richer context for consideration and discussion. It would be more meaningful to examine how students struggled and grew.

Finally, at the end of the intervention sessions, the post-test was conducted with the same test items as in the pre-test and marked following a scoring rubric. Then, the post-test scores from both groups were collected to identify the analysis result.

### 3.2.6 Data Analysis

After consulting with experts and peers, the student responses were marked by teachers and verified by the researcher following a developed scoring rubric (**see Appendix 8**). Responses were categorized as “no response” when there is no attempt to answer the question, and marked as “0”; “incorrect” when the answer provided is completely wrong, and marked as “1”; “partly correct” when the answer shows some understanding of the concept but is incomplete or contains minor errors, and marked as “2”; and “correct” when the answer is completely correct and demonstrates a clear understanding of the concept, and marked as “3.” Each test item has 3 points for VR and 3 points for CU, for a total of 6 points. An example of a scoring rubric is shown in Table 22.

Table 22. The scoring rubric of Test Item Number 1

Test items and objectives	Response	VR	CU	Scoring
1. Set four points, which are given as A(3,2), B(-2,1), C(4,-1) and D(-1,-2), in the figure below and explain how you get the answer.  <b>Objective:</b> <b>VR</b> To identify the students' abilities: <ul style="list-style-type: none"> <li>- to identify and interpret visual information, including visual,</li> </ul>	No answer	There is no attempt to answer the question	There is no attempt to answer the question	0
	Incorrect	- The answer provided is completely incorrect in response to the problem given. - Students can plot only one point among the four points of a given problem (Recognize). 	- The answer provided is completely incorrect when performing the mathematical procedure (Reason). - Students can explain only one of the four points of their procedures (Reason).  The reason that I draw a diagram like this is that the coordinates tell me the value for plotting these points. 	1

<p>symbolic, and verbal representations (Recognition);</p> <ul style="list-style-type: none"> <li>- to transform and coordinate different types of picture representations in mind (Mental Operation); and</li> <li>- to create geometrical drawings to clarify the problem (External Representation).</li> </ul> <p><b>Conceptual understanding</b></p> <p>To describe the students' abilities to</p> <ul style="list-style-type: none"> <li>- explain the mathematical concept following the mathematical procedure, including calculation (Properties);</li> <li>- provide the justifications for the mathematical procedure (Reason), and</li> <li>- use the mathematical expression to give the reason in relation to the visual representation (Representation).</li> </ul>			<p>The reason that I draw a diagram like this is that the coordinates tell me the value for plotting these points.</p>		
	Partly correct	<ul style="list-style-type: none"> <li>- The answer shows some understanding of drawing the pictures but is incomplete or contains minor errors (External representation).</li> <li>- Students can plot or construct more than the average of the correct answer and they do not show the symbol of those points (Recognize).</li> </ul>	<ul style="list-style-type: none"> <li>- The answer shows some of the correct procedures for using mathematical expressions (symbols and calculations) (Representation).</li> <li>- Students can explain the procedure for more than two or three points out of the four points (Reason)</li> </ul> <p> </p> <ul style="list-style-type: none"> <li>- A(3,2) combined by three units along the x-axis and two units along the y-axis</li> <li>- B(-2,1) combined by -2 units along the x-axis and 1 unit along the y-axis</li> <li>- C(4,1) combined by four units along the x-axis and 1 unit along the y-axis.</li> </ul> <p> </p> <p> </p> <p> </p> <p> </p>	<p> </p> <ul style="list-style-type: none"> <li>. A, because all values of A are positive.</li> <li>. B, because B has a negative value on the x-axis.</li> <li>. C, because B has a negative value on the y-axis</li> <li>. D, because all values of D are negative.</li> </ul>	2
	Correct	<ul style="list-style-type: none"> <li>- Students draw the picture clearly in responding to the</li> </ul>	<ul style="list-style-type: none"> <li>- Students can plot all four points correctly by given problem (Properties)</li> </ul>	3	

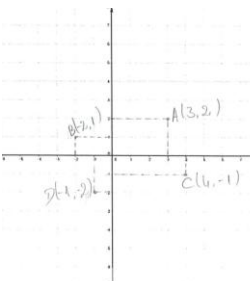
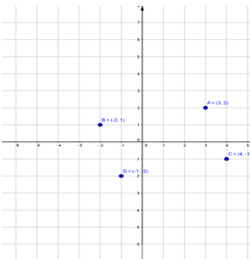
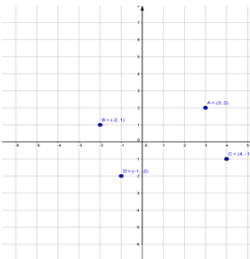
		<p>problems (External representation).</p> <ul style="list-style-type: none"> <li>- Use skill in drawing the picture to make it easy to understand (Mental representation).</li> <li>- Students use symbols to clarify the picture in responding to the given problem (Recognize).</li> </ul>	<ul style="list-style-type: none"> <li>- Students can explain the following mathematical procedure, provide a correct reason through the procedure, and, explain in the right way and use geometrical expressions to give the reason (Reason).</li> <li>- Students able to draw a picture in responding to given problem and easy to identify the shape (Representation).</li> </ul>	
			<p>- ចំពោះ A(3,2) បានមកពីការផ្តោតលើអ័ក្ស x គឺស្មើ 3 និង អ័ក្ស y គឺស្មើ 2  - ចំពោះ B(-2,1) បានមកពីការផ្តោតលើអ័ក្ស x គឺស្មើ -2 និង អ័ក្ស y គឺស្មើ 1  - ចំពោះ C(4,-1) បានមកពីការផ្តោតលើអ័ក្ស x គឺស្មើ 4 និង អ័ក្ស y គឺស្មើ -1  - ចំពោះ D(-1,-2) បានមកពីការផ្តោតលើអ័ក្ស x គឺស្មើ -1 និង អ័ក្ស y គឺស្មើ -2</p>	
		<p>. Expected answer</p> 	<ul style="list-style-type: none"> <li>. Point A(3,2) came from the x-axis equal to 3 and the y-axis equal to 2.</li> <li>. Point B(-2,1) came from the x-axis equal to -2 and the y-axis equal to 1.</li> <li>. Point C(4,-1) came from the x-axis equal to 4 and the y-axis equal to -1.</li> <li>. Point D(-1,-2) came from the x-axis equal to -1 and the y-axis equal to -2.</li> </ul>	
		<p>. Expected answer</p> 	<p>. Expected answer</p> <p>By the given problem, we can set the points below:</p> <ul style="list-style-type: none"> <li>- Point A(3,2) at the first quadrant because <math>x_A = 3 &gt; 0</math> and <math>y_A = 2 &gt; 0</math></li> <li>- Point B(-2,1) at the second quadrant because <math>x_B = -2 &lt; 0</math> and <math>y_B = 1 &gt; 0</math></li> <li>- Point C(-4,-1) at the third quadrant because <math>x_C = 4 &gt; 0</math> and <math>y_C = -1 &lt; 0</math></li> <li>- Point D(-1,-2) at the fourth quadrant because <math>x_D = -1 &lt; 0</math> and <math>y_D = -2 &lt; 0</math></li> </ul> <p>Figure:</p> <ul style="list-style-type: none"> <li>- Set the four points at the right position</li> </ul>	

Table 22 shows examples of the student responses in both VR and CU. A student who responded correctly achieved three points for each test item. The total scores of the five test items were 15 for VR and another 15 for CU.

1. The data analysis was drawn from student responses following this scoring rubric. IBM SPSS Statistics Data Editor version 27 was used to analyze the data, which was conducted to describe the students' abilities in solving vector problems related to VR and CU.
2. Describe the students' data through the intervention. In this step, we describe the results by determining the descriptive statistics, namely the measurement of central tendency and the size of data distribution, focusing on the skewness value.
3. Identify students' ability to solve geometry problems through VR and CU. In this step, we investigate the students' responses in both the pre-test and the post-test results to assess their improvement in each test item through the comparison.
4. Investigate students' VR and CU through intervention. In this step, we assess the difference between the pre-test and post-test by analyzing the Mean Ranks in students' understanding of VR and CU, using the Mann–Whitney U-test, a nonparametric test because the two data groups were not normally distributed (Morgan et al., 2011). This analysis evaluated the strength of the relationship (effect size) (Cohen, 1988). Although an effect size measure is not provided in the output, it is easy to compute one for the Mann–Whitney U-test “ $r$ ” from Standardized Test Statistics “ $z$ ” information provided in the test statistic table using the conversion formula  $r = \frac{z}{\sqrt{N}}$ . The comparison of  $r$  effect size was classified as shown in Table 23.

Table 23. The classification of a relationship (effect size)

The general interpretation of the strength of a relationship	r  effect size
Much larger than the typical	r  ≥ 70
Large or larger than the typical	> 50
Medium or typical	> 30
Small or smaller than the typical	< 10

Source: Interpretation of the strength of a relationship (effect size) (Morgan et al., 2011, p. 101).

5. Differentiate between post-test results using the **Normalized Gain Score** (Hake, 1998). In this step, we investigate vector operations and their effects. It is important to note that both groups are at equal levels at the beginning of the intervention. This calculation can be done to confirm how much the students gain knowledge after the intervention from the pre-test to the post-test (or maximum possible gain). The normalized gain score formula is shown below:

$$\text{Normalized Gain score} = \frac{\text{Posttest Score} - \text{Pretest Score}}{\text{Total\_Max\_Score} - \text{Pretest Score}}$$

The gain value is used to quantify how much the learners have improved relative to the maximum possible improvement they could have achieved. The percentages and categories of the normalized gain scores are shown in Table 24.

Table 24. The categories and percentages of the normalized gain scores

Category	Percentage (%)
Not effective	<40
Less effective	40 < 56
Effective enough	56 < 75
Effective	> 75

Source: Adapted from Hake (1998).

6. Examine the relationship between VR and CU. As the data are not normally distributed, **Spearman's rho, a Nonparametric Correlation** statistic, will be used to examine the relationship. The pre-test and post-test results will be used to

identify the correlation between the two aspects. The criteria used to identify the relationship between the test items are shown in Table 25.

*Table 25. Strength of correlation criteria*

<b>Amount of <math>r_s</math></b>	<b>Strength of correlation</b>
0.0 < 0.1	No correlation
0.1 < 0.3	Low correlation
0.3 < 0.5	Moderate correlation
0.5 < 0.7	High correlation
0.7 > 1	Very high correlation

Source: Kuckartz et al. (2013, p. 213).

Lastly, the data analysis of the students' interviews was conducted. Coding was done against data collected from groups of participants, such as curriculum developers, teachers, and students, to describe the improvement points and supplement the results. The responses were coded as shown in Table 26.

*Table 26. Response Coding for the results of the interviewees.*

<b>Type of respondents</b>	<b>Categories</b>	<b>Code</b>
	Management	CD1
Curriculum Developers	Management	CD2
	Management	CD3
	Teaching	TCP1
Teachers	Teaching	TCB1
	Teaching	TCB1
Students	Experimental group	STEx1 to 100
	Control Group	STC01 to 68

Moreover, data from 12 interviewed students were categorized and coded differently for qualitative data analysis. The student scores were classified as low, medium, or high. This selection was made based on the student's pre-test achievement score. Two students were selected from the lowest scores of the students from the control group, and the other two from the experimental group. We used the same selection method to select students from medium and high-achievement test scores. Table 27 shows the codes for the interviewed students.

Table 27. Interviewed students for qualitative analysis.

Type of respondents	Categories	Code	Pre-Test Scores
Students	High achievement test scores	CoH1	14.00
		CoH2	14.00
		ExH1	21.00
	Medium achievement test score	ExH2	17.00
		CoM1	7.00
		CoM2	7.00
	Low achievement test score	ExM1	15.00
		ExM2	14.00
		CoL1	2.00
		CoL2	1.00
		ExL1	1.00
		ExL2	2.00

### 3.2.7 Ethical Consideration

The author was aware of the importance of ethical considerations. Before starting the experimental study, the author applied for ethical permission from the Ethical Committee of Hiroshima University as part of the field survey. It took approximately three weeks before the committee approved the application (**Appendix 13**). The author brought a permission letter from MoEYS to both schools for data collection (**Appendix 14**).

The purpose and procedure of the study were included in the application forms for the field survey in the target schools. This study took care to protect the rights and privacy of all participants. The subjects were also informed of the purpose and procedure followed in this study in the consent forms. The participants (curriculum developers, teachers, and students) volunteered to participate in the study. No incentives were provided. Participants' anonymity and confidentiality were ensured through pseudonymization.

Students in the control and experimental groups were requested to provide the same treatment after the data collection period. In short, the researcher was fully aware of the significance of the research's ethical considerations and followed all ethical application guidelines before approaching the target school.

## CHAPTER 4: RESULTS

This chapter presents the results of the study.

### 4.1 RESULT OF CURRICULUM ANALYSIS

Although this research focused on upper secondary education, the curriculum analysis was conducted from primary to upper secondary education. The analysis follows:

#### 4.1.1 Arrangement of Domain, Sub-domain, and Contents

Regarding the arrangement of the geometry domain, sub-domains, and the contents of geometry, the results show that five sub-domains and 68 contents have been identified under the geometry domain, as shown in Table 28.

*Table 28. The arrangement of the geometry domain, sub-domains, and contents from grades 1 to 12*

Sub-domains and contents in the geometry domain	Grades											
	1	2	3	4	5	6	7	8	9	10	11	12
<b>Plane geometry</b>												
Points, lines, and curves	■											
Patterns	■											
Two-dimensional geometric shapes		■						■				
Rectangles and triangles			■									
Area and perimeter			■	■		■						
Angles				■	■		■					
Squares and rectangles				■								
Construction geometry					■							
Area of a triangle					■							
Circles						■						
The foundation of geometric shapes							■					
Perimeter and area of a polygon							■					
Perimeter and area of a circle							■					
Rectangles								■				
Polygons								■				
Area of a rectangle								■				

Position of circles and lines					■	
A line and a special segment intersect in a triangle.					■	
Circles and lines					■	
Properties of angles within a circle and regular polygons					■	
Thales' theorem					■	
Parabolas						■
Ellipse						■
Hyperbola						■
<b>Solid geometry</b>						
Geometric shapes	■					
Three-dimensional geometric shapes		■				
Volume of a solid			■	■	■	■
Solids				■		
The surface area of a solid					■	■
Geometric shapes and planes in space						■
Surfaces of similar shape						■
The volume of a similar solid						■
Geometric shapes in space						■
Lines and parallel planes in space						■
Orthogonal in space						■
<b>Relations and transformations</b>						
Perpendicular and parallel lines		■				
Symmetric shapes		■				
Puzzle (Tessellation)		■				
Properties of triangles and rectangles			■	■		
Parallel lines and perpendicular lines					■	
Symmetries of a point					■	
Symmetries of lines and planes					■	
Equilateral triangles					■	
Similarity of triangles						■
Moving transformation						■
Symmetric transformation						■
Rotations transformation						■
Homotheties transformation						■

<b>Construction</b>				
Plane coordinates (cartesian plane)	■			
Construction of mediators, bisectors of angles and triangles	■			
Construction of rectangles	■			
Scale		■		
Pythagorean theorem		■		
The trigonometric ratio in a triangle		■		
Triangle inequalities			■	
Distance between two points			■	
Linear equations		■	■	
Coordinate points in a plane			■	
Equation of a circle			■	
Solving inequation by graph			■	
Equation of a plane				■
Linear equation in space				■
Equation of a sphere				■
<b>Vector</b>				
Vectors and operations on vectors		■		
Practice with vectors		■		
Vector spaces				■
Scalar product of two vectors				■
Scalar product of two vector spaces				■

Sources: Detailed Mathematics Syllabus of Grades 1–12 (Sieng & Baba, 2024)

Note: Bold font indicates sub-domains, non-bold font indicates contents, and “■” indicates the grade for the content.

Table 28 shows that the Cambodian mathematics syllabi in the geometry domain consist of more content compared to international mathematics curricula (NIEI & PRI, 2021). Besides, these contents are neither well-organized nor well-sequenced. For example, the contents of “angle” appeared in grades 4, 5, and 7 when detailed information was examined at each grade. In grade 4, the content concerns how to construct acute and perpendicular angles using a protractor and lines, and it involves practice measuring the angle within eight directions. In grade 5, the content consists of the use of a protractor to construct straight angles, full rotation angles, and vertical

angles. In grade 7, the content concerns the notion of an angle, types of angles, measuring angles, and the use of a protractor to measure the angle by constructing it with a line, a compass, and perpendicular lines. This topic (i.e., “angle”) is skipped in grade 6 without a proper explanation. Therefore, this curriculum has weak coherence as a whole because some contents are skipped and repeated.

This kind of analysis is conducted based on the GTTM procedure (Schmidt et al., 2005a) and yields the following results regarding the Cambodian syllabi for geometry as a whole.

#### 4.1.2 Relation of Sub-Domains and Contents

The sub-domains and contents are extracted from the syllabi, and the grade levels in which the contents of the sub-domains are addressed in each grade level are indicated with the symbol “■.” Table 29 shows an example of the sub-domain of solid geometry and its contents.

Table 29. The relation between a sub-domain and its contents

<b>Sub-domain and contents</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
<b>Solid Geometry</b>												
Geometric shapes	■											
Three-dimensional geometric shapes		■				■						
The volume of a solid					■	■	■	■				
Solids						■						
The surface area of a solid							■	■				
Geometric shapes and planes in space								■				
Surfaces of similar shape										■		
The volume of a similar solid										■		
Geometric shapes in space											■	
Parallel lines and planes in space											■	
Orthogonality in space											■	

Note: Bold font indicates a sub-domain; non-bold font indicates contents; “■” indicates the grade for the contents.

Here, we can see the repetition of content, discontinuous treatment, and reverse treatment.

Regarding the repetition of contents, sub-domains and contents are repeated by simply adding superficial complexities to the contents without much difference in geometrical ideas from lower to upper grades.

Regarding discontinuous treatment or skipping the grades, an example is the content of the “three-dimensional geometric shape”. It is addressed in grade 2 and again in grade 6, meaning it is not continuously treated from grade to grade. Meanwhile, as for reverse treatment, the content of “solid” is addressed in grade 6, while the content of “volume of a solid” is addressed in grade 5. The content of “solid” should be introduced before the content of “volume of a solid.” In other words, to measure the solid, we have to know its shape first.

#### 4.1.3 Relation of Contents and Sub-contents Across Grades

The relation of contents and sub-contents across grades is repeated, and they are not sequenced or arranged in order, as shown in Table 30.

*Table 30. The relation between contents and sub-contents within and across grades*

<b>Grade 5</b>	<b>Grade 6</b>	<b>Grade 7</b>	<b>Grade 8</b>
<b>Volume of a Solid</b>	<b>Solid</b>	<b>Volume of a solid</b>	<b>Volume of a solid</b>
- Extension of a solid shape	- Notion of a solid	- Notion of a solid	
- Construction of a solid with a unit of cubes	- Notion of the surface area of a solid	- Change the unit of volume	- Solve problems related to the surface edge of combined solids
- Construction of Cubes	- Length of a cube edge		
- Measurement and measuring volume	<b>Volume of a solid</b>	- Volume of a right prism	- Volume of a right prism
- Volume of a cube and cuboid	- Unit of volume	- Volume of a cylinder	- Volume of a cylinder
- Volume of a liquid	- Volume of a solid		- Volume of a pyramid
	- Volume of a liquid		- Volume of a cone
	- Practice		- Volume of a sphere

Sources: Syllabi of Grades 1–9 (MoEYS, 2018a, 2018b)

Note: Bold font indicates contents, and non-bold font indicates sub-contents.

Table 30 presents the “volume of a solid” content and illustrates how its related contents and sub-contents are arranged from grades 5 to 8.

In grade 5, such sub-contents as “extension of a solid shape,” “construction of a solid with a unit of cubes,” and “construction of cubes” are introduced. The sub-content “extension of a solid shape” reflects a continuation of previously acquired knowledge and assumes that students already understand the basics of the “notion of a solid”. The subsequent sub-contents, including “construction of a solid with a unit of cubes” and “construction of cubes”, are central to the conceptual development of volume in this grade, particularly in helping students understand the structure and formation of three-dimensional shapes.

Additionally, three sub-contents, “measurement and measuring a volume”, “volume of a cube and cuboid”, and “volume of a liquid”, are related to measurement. These topics are organized in order of increasing difficulty, beginning with the understanding of a unit of measurement, followed by the application of that unit in simple contexts, and culminating in its extension to liquid volume. However, some content appears to be repeated unnecessarily. For instance, the sub-content “unit of volume” reappears, possibly by introducing new units, such as  $\text{km}^3$ ,  $\text{m}^3$ , and  $\text{mm}^3$ . Likewise, the sub-content “volume of a solid” is repeated across multiple grade levels, potentially covering different solid shapes beyond cubes and cuboids, yet the conceptual development from one grade to the next is not clearly articulated.

There are also sequencing issues in the curriculum. The sub-content “length of a cube edge” is introduced in grade 6, although this concept is required earlier in grade 5 when students are first expected to compute the volume of cubes and cuboids. Introducing this prerequisite knowledge only in a later grade may hinder students’ comprehension. More critically, the content “solid”, which includes fundamental concepts such as the “notion of a solid” and “surface of a solid”, is introduced in grade 6. However, these concepts are essential for understanding volume and should be taught before students encounter “volume of a solid” in grade 5.

Therefore, the curriculum exhibits several coherence-related issues. Some contents are presented in reverse order, such as introducing “solid” after the “volume of a solid,” while others are repeated across grades without clear advancement in conceptual complexity. These issues may hinder students’ ability to develop a solid

and connected understanding of volume, highlighting the need for a more logically sequenced and developmentally appropriate curriculum structure.

#### 4.1.4 Relation Between the Contents, Sub-contents, and Learning Outcomes

The contents, sub-contents, and learning outcomes do not correspond with each other. An example of the relation of sub-contents and learning outcomes is shown in Table 31.

*Table 31. The relation of content, sub-contents, and learning outcomes within the grade level.*

Grade	Content	Sub-contents	Learning outcomes
5	Volume of a solid	<ul style="list-style-type: none"> <li>- Extension of a Solid shape</li> <li>- Construction of a solid with a unit of cubes</li> <li>- Construction of cubes</li> <li>- Measurement and measuring a volume</li> <li>- Volume of a cube and cuboid</li> <li>- Volume of a liquid</li> </ul>	<ul style="list-style-type: none"> <li>- Cut or fold a shape to make various solids, such as a parallelepiped, cube, and cylinder.</li> <li>- Use the unit of the cube to find a rectangular parallelepiped and cube</li> <li>- Use a measuring cup with 1000 ml and a cube with a volume of 10 cm<sup>3</sup> to prove that <math>1l = 1000 ml = 1000cm^3</math> and <math>1l = 1 dm^3</math>.</li> <li>- Use the dimensions of a cubic or rectangular parallelepiped to find the volume of a liquid in a non-specific geometric object used in daily life.</li> </ul>

Sources: Syllabi of grades 1–6 (MoEYS, 2018e)

Table 31 shows an example of sub-contents and learning outcomes within the contents of “volume of a solid” in grade 5. Six sub-contents appear under the content “volume of a solid”, but the learning outcomes include only four points, indicating no direct correspondence between learning outcomes and sub-contents. Besides, it is difficult to identify the relation between the learning outcomes and the sub-contents because the descriptions of the learning outcomes are mixed. The first learning outcome seems to correspond with the first and second sub-contents, and the last learning outcome with the fifth and sixth sub-contents.

Another example of vector contents in grade 10 found that sub-contents and learning outcomes do not correspond to each other within grade 10, which is called incoherence. The table below shows the correspondence between the sub-content and learning outcomes of vector topics.

*Table 32. The corresponding contents, sub-contents and learning outcome of the vector within grade 10*

<b>Grade</b>	<b>Content</b>	<b>Sub-content</b>	<b>Learning outcomes</b>
10	Vectors and operations on vectors	<ul style="list-style-type: none"> <li>- Meaning of vectors</li> <li>- Addition and subtraction of vectors</li> <li>- Multiplication of a vector by a real number</li> <li>- Coordinate of a vector in the plane</li> <li>- Measurement of algebraic vector</li> <li>- Scalar product</li> </ul>	<ul style="list-style-type: none"> <li>- Find the equality vector and the opposite vector</li> <li>- Calculate the addition and subtraction of vectors</li> <li>- Calculate the multiplication of the vector by a real number and display the vector colinear</li> <li>- Calculate the coordinates of vectors</li> <li>- Calculate the scalar product of vectors</li> <li>- Practice solving problems related to daily life</li> </ul>

Table 32 shows that some sub-contents and learning outcomes do not correspond within the same grade 10. For example, the “measurement of algebraic vector” has no clear learning outcome stated in the curriculum. The learning outcome “practice solving the problem related to daily life” is not clearly stated which sub-contents does it belong to?

A learning outcome should be explicitly tied to the intended content and sub-contents because it can support textbook writers and teachers in unpacking the geometric content for writing the textbooks and planning the lesson (NIEI & PRI, 2021).

Therefore, the learning outcomes should be clearly identified to correspond to the sub-contents and contents.

#### 4.1.5 Relation of the Domain, Sub-domains, Contents, Sub-contents, and Learning Outcomes Across the Grades

In summary, in the analysis of geometry as a whole, the domain, sub-domains, contents, sub-contents, and learning outcomes do not correspond across grades and are not sequenced. In this case, we take an example of grades 5 and 6, as shown in Table 33.

Table 33. The relation of a domain, sub-domains, contents, sub-contents, and learning outcomes across the grades.

Grade	Domain	Sub-domain	Contents	Sub-contents	Learning outcome
5	Geometry	Solid geometry	Volume of a solid	<ul style="list-style-type: none"> <li>- Extension of a solid shape</li> <li>- Construction of a solid with a unit of cubes</li> <li>- Construction of cubes</li> <li>- Measurement and measuring a volume</li> <li>- Volume of a cube and cuboid</li> <li>- Volume of a liquid</li> </ul>	<ul style="list-style-type: none"> <li>- Cut or fold a shape to make various solids, such as a parallelepiped, cube, and cylinder</li> <li>- Use the unit of the cube to find the rectangular parallelepiped and cube</li> <li>- Use a measuring cup with <math>1000\text{ml}</math> and a cube with a volume of <math>10\text{cm}^3</math> to prove that <math>1\text{l} = 1000\text{ml} = 1000\text{cm}^3</math> and <math>1\text{l} = 1\text{dm}^3</math>.</li> </ul>

					- Use a cubic or rectangular parallelepiped by knowing its dimensions to find the volume of a liquid in a non-specific geometric container used in daily life.
6	Geometry	Solid geometry	Solid	<ul style="list-style-type: none"> <li>- Notion of a solid</li> <li>- Notion of the surface of a solid edge</li> </ul>	<ul style="list-style-type: none"> <li>- Define a solid by cutting a shape from paper and folding it.</li> <li>- Cut and fold paper to create solids, such as prisms, cubes, angles, cylinders, and pyramids.</li> </ul>
			Volume of a solid	<ul style="list-style-type: none"> <li>- Unit of volume</li> <li>- Volume of a solid</li> <li>- Length of the cube</li> <li>- Volume of a liquid</li> <li>- Practice</li> </ul>	<ul style="list-style-type: none"> <li>- Convert scale from small to large and vice versa.</li> <li>- Use the volume formula of a solid to calculate a rectangular parallelepiped and cylinder.</li> <li>- Find the length of the cubic edge from the surface base or its volume using the square root or cube root and a calculator.</li> <li>- Find an edge of a rectangular</li> </ul>

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parallelepiped by knowing the volume and two of its edges or the surface area of the base.

- Find volumes and the height of liquid in a container or rectangular parallelepiped.
- Use the volume formula of a cube, a rectangular parallelepiped, cylinders, and the volume of a liquid to solve problems related to daily life.

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Sources: Syllabi of grades 1–6 (MoEYS, 2018e) (MoEYS, 2018b)

Table 33 shows an example of the relation of a domain, sub-domains, contents, sub-contents, and learning outcomes across grades 5 and 6. Here, the content “solid” is addressed only in grade 6, while “volume of a solid” is addressed in grades 5 and 6. We consider that the content “solid” should be introduced before the content “volume of a solid,” because students have to learn the shapes before calculating their volumes. This means that “solid” should be taught in grade 5 and “volume of a solid” in grade 6. The learning outcome should be more specific in terms of what students are expected to understand and work with regarding various solids, including prisms, cones, pyramids, and cylinders. They are also expected to explore the relationships between the dimensions, perimeter area, and volume of different shapes and solids.

Another example is vector contents allocated in the Cambodian mathematics curriculum for upper secondary schools. It was also found that there is no connection between grade 10 and grade 12. The table below shows the vector contents of the Cambodian mathematics curriculum for upper secondary education.

Table 34. Geometry contents are allocated in upper secondary education in Cambodia.

Grade 10		Grade 11		Grade 12	
Science	Social Science	Science	Social Science	Science	Social Science
Coordinate of point in plane	Coordinate of point in plane	Geometry shape in space	<i>There is no geometry content in this grade for the social science track.</i>	Parabola	<i>There is no geometry content in this grade for the social science track.</i>
Linear equation	Linear equation	Line and parallel plane in space		Ellipse	
Circular equation	Circular equation	Orthogonality in space		Hyperbola	
Vector and operation on vectors	Vector and operation on vectors			Vector space	
Practice of the vectors	Practice of the vectors			Scalar product of two vector	
Translated transformation				Scalar product of two vector space	
Reflected transformation				Plane equation	
Rotiated transformatio				Line equation in space	
Homothetied transformation				Sphere equation	

Source: (MoEYS, 2018f, 2018g)

Table 34 shows that no vector contents are allocated in grade 11, indicating the disconnection between the grades. This connection is very important so that students can continue learning this content from grade to grade with more advanced cognitive skills.

In summary, the results of the analysis indicate that the detailed mathematics syllabi on geometry lack coherence owing to (1) repetition of some sub-domains and contents, (2) contents and sub-contents are not sequenced across grades, (3) contents, sub-contents, and learning outcomes do not correspond within a given grade, and (4) the domain, sub-domains, contents, sub-contents, and learning outcomes do not correspond across grades. Therefore, the curriculum, which lacks coherence, can lead to teachers' difficulty in organising the teaching materials and students' difficulty in learning because the relationship between the content, sub-contents, and learning outcomes is disconnected and makes the students lack conceptual understanding of subjects. A lack of coherence makes it hard for students to connect mathematical procedures with reasoning or real-world applications, reducing their ability to solve geometry problems and higher-order thinking skills (NIEI & PRI, 2021).

#### 4.1.6 Field Survey Interview with a Focus on Curriculum

Against the result of the curriculum analysis, the researcher tried to identify how the curriculum developers considered the issues. This section presents the results of

the interview with such curriculum developers. The interviews included reference materials, technology integration, and the curriculum's challenges and improvements. Even though no words are stated regarding curriculum coherence, based on the definition of curriculum coherence in this study, the interview interaction was discussed.

Three curriculum developers were interviewed to reflect on the design and implementation of the curriculum. The semi-structured interview guide was administered individually. The results were described as follows:

**Noted.** *Re= researcher; CD= Curriculum developer*

### **Interview results of three curriculum developers**

...

**Re. Did you refer to some curricula from other countries during the drafting process for curriculum development? If yes, which countries?**

**CD1.** *Yes, we have integrated with other countries, such as Singapore, Thailand, Malaysia, and other developed countries, such as the United Kingdom and Australia.*

**CD2.** *During the curriculum revision draft, we also collaborated with other countries through materials and some guidance from experts indirectly, with approximately 30% from the region (Thailand, Vietnam, Singapore) and around 10% from other regions (America, Japan, France).*

**CD3.** *Yes, of course, we must learn about other countries' curricula and developed countries in the world, such as Singapore, Japan, France, and America, to integrate the relevant subjects in response to the national policy on developing human resources.*

**Re. What criteria are used to set the objectives of the mathematics curriculum for each lesson? Specifically for geometry content?**

**CD1.** *I am not sure.*

**CD2.** *The strategies for identifying the objectives of the mathematics curriculum based on each content area, such as those related to daily life, involve developing questions and exercises in relation to regional and international competitions in response to the current situation.*

**CD3.** *The strategies include (1) identifying the knowledge, skills, and attitudes, (2) using digital technologies in teaching and learning, and (3) implementing student-centered approaches.*

**Re. What challenges does the current mathematics curriculum present to teachers and students in schools?**

**CD1.** *The curriculum is too lengthy for some content, and teachers cannot cover all the content within a year for each grade.*

**CD2.** *The current mathematics curriculum faces some challenges for both teachers and students in schools, such as (1) not being able to implement all content at each grade level, (2) implementation not meeting national standards, and (3) the MoEYS not distributing it nationwide, which causes teachers not to implement the new curriculum.*

**CD3.** *There are several challenges, such as (1) the contents are not modernized due to a lack of budget for revision, (2) too much content that teachers cannot complete in one year, (3) very few elements integrating technology, (4) insufficient documentation, drills, and teacher guides for support, (5) too much of theory and a lack of practical application, and (6) a shortage of teachers who can use IT in their subjects.*

**Re. Does the current mathematics curriculum emphasize the integration of technology? If so, how?**

**CD1.** *Yes. It does. For examples, they have been integrated into the content such as drawing the pictures.*

**CD2.** *Very limited.*

**CD3.** *It was limited integration based on the ability of the teaching material, such as a calculator for students, but for teachers, based on the existing resources and teachers' integration ability.*

Through the interview in the conversation above, the Cambodian mathematics curriculum integrates elements from various international education systems, particularly from regional and developed countries, to align with national policies on human resource development. However, the curriculum faces challenges in implementation, including too much content, outdated materials, limited technological integration, and inconsistent nationwide distribution. While efforts have been made to incorporate digital tools and student-centered approaches, technological integration remains minimal due to resource constraints and limited teacher training. Addressing these challenges through curriculum modernization, improved resource allocation,

and enhanced teacher support is essential for achieving a more effective and globally competitive mathematics education system.

Furthermore, in terms of the perception of the curriculum developers and teachers, the results of interviews were conducted, and the results indicated as follows:

**Noted.** *Re= researcher; CD= Curriculum developer; TCP1=Teacher 1, TCB1=Teacher 2*

...

**Re.** **GeoGebra is a software that can support mathematics teaching and learning. Do you think GeoGebra should be integrated into the mathematics curriculum, particularly for geometry content? If so, how should it be integrated?**

**CD1.** *"...It should be included in the training for existing teachers and in teacher education programs..."*

**CD2.** *"...We are happy to integrate GeoGebra software into the mathematics curriculum, particularly in geometry content, in response to the new generation of artificial intelligence (AI)..."*

**CD3.** *"... In the context of modern education in the 21st century, it is important to integrate contemporary practices into the mathematics curriculum and textbooks. This integration will provide guidance for both students and teachers on how to effectively apply these practices in real-world scenarios, making it easier to understand mathematical concepts..."*

**TCP1.** *"...It should be integrated from grade 10 because it can help students explore space geometry lessons, plan equations, and tangent equations to see reality using computers..."*

**TCB1.** *"...GeoGebra software should be integrated into the mathematics curriculum, as well as in teaching and learning materials. For example, it can be used to illustrate how to create diagrams, draw pictures, and demonstrate concepts in spatial geometry. This integration makes it easier for students to understand mathematical concepts..."*

**Re.** **Do you have any suggestions for improving the mathematics curriculum, particularly in geometry, to achieve better results?**

**CD1.** *‘...It should focus more on practice than theory to help students see future directions...’.*

**CD2.** *“...To improve our mathematics curriculum, we should involve experts in the field and utilize modern documentation from developed countries for further revisions. The Ministry of Education, Youth and Sport (MoEYS) needs to increase the budget allocated for curriculum development. Additionally, it's important to provide training for teachers and specifically for those in charge of implementing the curriculum. Facilitating study visits to observe how developed countries execute their curriculum could also be beneficial...”.*

**CD3.** *“...Develop the next revision as a standard that aligns with the curriculum both regionally and globally. It is important to integrate GeoGebra software or other effective applications to support digital education...”.*

**TCP1.** *“...It should be integrated not only into the curriculum and textbooks but also into teacher training programs such as BA+1 and BA+2. This integration will support student teachers in fully acquiring the ability to use GeoGebra software as an effective teaching and learning tool...”.*

**TCB1.** *“...It is important to integrate information technology into teaching and learning, particularly using GeoGebra software to enhance students' problem-solving abilities in mathematics. This approach can help students understand the value of learning geometry in daily life through demonstrations of spatial geometry concepts...”.*

Through the interview, the perception of curriculum developers and teachers was addressed that integrating GeoGebra into the Cambodian mathematics curriculum, particularly in geometry, is widely supported as it aligns with modern educational practices and enhances students' understanding of spatial concepts. They recommended that GeoGebra software be included in teacher training programs and in-service teacher education to ensure effective implementation. They are highly supportive and committed to integrating GeoGebra software into their geometry teaching, as well as in other subjects. They noted that the software effectively helps them illustrate the connection between prior knowledge and mathematical concepts, particularly in geometry, which can be challenging for students to grasp. The software should be introduced from grade 10 onwards, where it can facilitate exploration of space geometry, equations, and real-world applications. Additionally, incorporating GeoGebra software into textbooks and teaching materials will provide students with

hands-on experience in constructing diagrams, visualizing geometric concepts, and improving problem-solving skills.

A stronger emphasis on practical applications over theoretical instruction is necessary to further enhance the mathematics curriculum, particularly in geometry. Key recommendations include engaging experts in curriculum development, adopting best practices from developed countries, and increasing financial support for curriculum reform. Additionally, aligning curriculum revisions with both regional and global educational standards will ensure their relevance. Training programs like Bachelor +1 (BA+1)<sup>4</sup> and Bachelor +2 (BA+2)<sup>5</sup> should incorporate GeoGebra software to equip future educators with the necessary digital competencies. Finally, integrating information technology into teaching and learning, particularly through GeoGebra software, can significantly enhance students' mathematical understanding and application in real-life contexts.

## **4.2 RESULT OF THE COMPARATIVE ANALYSIS OF THE CURRICULUM**

We obtained the comparative analysis of the mathematics curriculum of Cambodia and Singapore by following the steps described in the methodology section.

### **4.2.1 Constructed Curriculum Structure**

There are different perspectives on developing the curriculum framework for both countries, Cambodia and Singapore. The analysis reveals that the Cambodia Mathematics Curriculum (CMC) framework aims to improve the quality of education through the knowledge, skills, and attitudes necessary to thrive in daily life and further studies. The framework seeks to ensure students develop their full potential, enabling them to contribute to national development and integrate into the global community. It

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<sup>4</sup> The BA+1 programme exists in the National Institute of education (NIE) which mean students are required to graduate from various universities with a bachelor's degree and do the entrance examination to become a government employee. After they graduate from the NIE, they obtain a certificate as a teacher with higher education.

<sup>5</sup> The BA+2 programme exists in the NIE which mean students are required to graduate from various universities with a bachelor's degree and do the entrance examination to become a government employee. After they graduate from the NIE, they obtain a certificate as a teacher with higher education with the equivalence of a master's degree of education.

includes fostering competencies in areas such as literacy, numeracy, critical thinking, and the ability to apply knowledge in practical contexts, all in alignment with Cambodia’s goal of becoming an upper-middle-income country by the year 2030 and a high-income country by the year 2050. The Singapore Mathematics Curriculum Framework (SMCF) aim to improve mathematics education in different socio-cultural contexts, emphasizing content and learning processes in various subjects with a focus on student learning experiences.

Against the findings of the Cambodian curriculum from Section 4.1, the core competencies of the Singapore Mathematics Curriculum (SMC) are coherently designed to equip students with essential skills for the 21st century, including mathematical concepts, skills, processes, metacognition, and attitudes. These interrelated competencies support students in applying mathematics knowledge and skills for future changes in various fields. The summary of the differences in the curriculum framework of both countries is shown in Table 35.

*Table 35. Summary of the constructed curriculum structures for Cambodia and Singapore*

<b>Cambodia</b>	<b>Singapore</b>
<ul style="list-style-type: none"> <li>- Focus on developing students’ awareness of how mathematics is used in their own and other communities.</li> <li>- Stress on analyzing a mathematical concept, such as logical reasoning</li> <li>- Emphasis on applying the learning outcomes with a focus on knowledge, skills, and attitude</li> </ul>	<ul style="list-style-type: none"> <li>- Emphasizes a conceptual and ideological approach</li> <li>- In-depth CU, skills, proficiency, and mathematical procedure, and emphasis on attitudes and metacognition</li> <li>- Build a strong foundation in mathematical concepts based on student learning experiences, mainly focusing on problem-solving skills.</li> </ul>

Table 35 highlights that both Cambodia and Singapore value a comprehensive approach to mathematics education, although their methods and emphases differ, with Cambodia focusing more on practical application and awareness and Singapore on deep CU and problem-solving abilities.

## 4.2.2 Comparison of Sub-Domains and Contents

There is an arrangement of sub-domains and contents extracted from the syllabi. The corresponding grade level is indicated with the symbol “■”. The sub-domain of plane geometry is taken as an example to compare the contents between the countries (see Table 36).

Table 36. Example of a comparison of sub-domains and contents

Cambodia	Plane geometry	1	2	3	4	5	6	7	8	9	10	11	12
	Point, line, and curve	■											
	pattern	■											
	Two-dimensional geometry shape		■						■				
	Rectangles and triangles			■									
	Area and perimeter			■	■		■						
	Angle				■	■		■					
	Square and rectangle				■								
	Construction geometry					■							
	Area of a triangle					■							
	Foundation of geometric shape							■					
	Perimeter and the area of a polygon							■					
	Rectangle								■				
	Polygon								■				
	Area of rectangular								■				
	circle						■						
	Perimeter and area of a circle							■					
	Circle and position between a circle and a line								■				
	Line and special segment meet inner the triangle.								■				
	Circle and line									■			

	Properties of the angle of the circle and regular polygon				■
	Thales' theorem				■
<b>Singapore</b>	<b>Plane geometry</b>				
	2D shapes	■	■		
	Perpendicular and parallel lines			■	
	Area and perimeter		■	■	■
	Angles		■	■	■
	Rectangle and square			■	
	Triangle				■
	Area of the triangle				■
	Parallelogram, rhombus, and trapezium				■
	Area and circumference of a circle				■
	Angles, triangles and polygons				■ ■
	Pythagoras' theorem and trigonometry				■
	Mensuration				■

**Note:** Bold font indicates a sub-domain; non-bold font indicates contents; “■” indicates the grade for the contents.

**Source.** The author developed the table by examining the curricula from both countries.

Table 36 illustrates a comparative analysis of the Cambodian and Singaporean mathematics curricula, considering the plane geometry domain. As mentioned in Section 4.1, the analysis of the Cambodian curriculum shows that the relations between some sub-domains and contents are not consistently addressed or are repeated by simply adding superficial complexities to the contents without much difference in geometrical ideas from lower to upper grades. An example of non-continuous treatment is the “two-dimensional geometry shape” content. It was

addressed in grade 2 and again in grade 7, meaning that it is not continuously treated from grade to grade. Similarly, “area and parameter” and “angle” are not continuously treated from grade to grade.

On the other hand, the Singaporean curriculum is designed and addressed coherently and continuously from grade to grade. For example, the contents of “area and parameter” and “angle” develop significantly with the same contents. Moreover, the whole curriculum has less content than the Cambodian curriculum, and student learning experiences are based on the content. The curriculum shows a structured progression of geometric concepts, beginning with basic shapes in early grades and gradually introducing more complex topics as students advance. There is a clear emphasis on foundational concepts like 2D shapes, area, and perimeter in the early grades, which serve as the basis for more complex topics like trigonometry and measurement in later grades. The curriculum appears to be designed to build upon students’ knowledge incrementally, ensuring that by the time they reach higher grades, they have a solid understanding of the basics, enabling them to tackle more challenging topics.

#### 4.2.3 Comparison of Contents, Sub-contents, and Learning Outcomes Between Mathematics Curricula of Both Countries

The results of the analysis of both curricula highlight that different structures are apparent in each of them. An example of such differences is shown in Table 37.

*Table 37. An example of the mathematics curriculum of both countries for grade 1*

Cambodia	Contents	Sub-contents	Learning outcomes
	- Point, Line, necklace	- Point and line - Curve and neckless (not a straight line)	- Set a point, draw a line, curve, and necklace, and connect the point to create a line and another shape.
	- Geometry shape	- Set a point, then draw a line, curve, and necklace. Connect the points to create a line and another shape.	- Draw a two-dimensional geometry shape that has four sides by using a line, and draw a circle by using an object as a circle  - Compare and describe the size of things that they used to know as

			<p>two-dimensional (e.g., a Whiteboard is a rectangle)</p> <ul style="list-style-type: none"> <li>- Compare and describe the size of things that they used to know as two-dimensional (e.g., a box of chalk, Cubes)</li> <li>- Draw shapes and geometry at a point or by using a sample that has been given.</li> <li>- Fill the pattern by size and color, and create samples by size and color.</li> </ul>
	- Patterns	<ul style="list-style-type: none"> <li>- Patterns by shape</li> <li>- Patterns by size</li> <li>- Patterns by color</li> </ul>	<ul style="list-style-type: none"> <li>- Fill the pattern by shape, size, and color</li> <li>- Create a sample by shape, size, color</li> </ul>
Singapore	2D Shapes	<p>1.1) Identifying, naming, describing, and classifying 2D shapes</p> <ul style="list-style-type: none"> <li>• Rectangle</li> <li>• Square</li> <li>• Circle</li> <li>• Triangle</li> </ul> <p>1.2) Making/completing patterns with 2D shapes according to one or two of the following attributes</p> <ul style="list-style-type: none"> <li>• Size</li> <li>• Shape</li> <li>• Color</li> <li>• Orientation</li> </ul>	<p>a) Recognize, name and describe the four basic 2D shapes (rectangle, square, circle and triangle) from real objects and pictures (drawings and photographs).</p> <p>b) Trace the outline of 2D shapes from 3D objects.</p> <p>c) Identify and describe 2D shapes in different sizes and orientations.</p> <p>d) Form a 2D shape from cut-out pieces of the shape.</p> <p>e) Guess a 2D shape from a description of the shape</p> <p>f) Recognize and describe the differences/similarities between two 2D shapes according to attributes such as sides, corners, sizes, and colors.</p>

- 
- g) Work in groups to sort 2D shapes in different ways and explain how the shapes are sorted.
  - h) Use 2D shapes or applets to create patterns according to one or two attributes (size, shape, color, and orientation) and describe the patterns,
  - i) Work in groups to create a pattern and then invite other groups to guess the missing shape(s) in the pattern and explain the pattern.
- 

*Source: The author developed the table based on the mathematics syllabi of both countries.*

Table 37 indicates that there is more geometry content in the first grade of the CMC than in the SMC, and students are required to have in-depth thinking skills to understand this content. The sub-contents depend heavily on the main content. However, the sub-contents and learning outcomes do not correspond to each other, as it is unclear which learning outcomes align with specific sub-contents and contents. For example, the content of “point, line, and curve” involves three geometrical shapes, but the learning outcome is addressed in only one paragraph, combining all three. This can confuse teachers when preparing teaching materials, and textbook writers may struggle to distinguish between the aspects.

In contrast, at the same grade level, the SMC contains less content than the CMC. For instance, the sub-contents listed in Table 37 cover the knowledge of 2D shapes. The learning outcomes are structured as steps in the learning trajectory, progressing from one step to the next. This approach can help teachers develop teaching materials more easily and guide textbook writers through each step.

In summary, the analysis shows that the CMC is still not fully coherent compared to the SMC (Grey, 2020). Singaporean students have achieved high performance in mathematics. The curriculum is not the single reason for high performance. However, the CMC developer should learn from it on how to enhance the curriculum’s existing spiral approach by strengthening the connections among

different contents, sub-contents, and learning outcomes (Hairon, 2021; NIEI & PRI, 2021). It is essential to clearly define objectives, contents, sub-contents, and learning outcomes and to investigate their interrelations thoroughly.

Additionally, for the curriculum to be effective, it must provide comprehensive coverage of essential skills and knowledge, ensuring that the selected content is age-appropriate. It should also have clarity regarding learning outcomes, specifying the concepts, conventions, techniques, results, and geometry processes intended for each content. In this sense, the curriculum should ensure that design, structure, and coherence, as well as the relation to domain, sub-domains, contents, sub-contents, and learning outcomes, are maintained within and across years.

### 4.3 RESULTS OF FIELD SURVEY.

Through the analysis of the students' responses, we have obtained the following results.

#### 4.3.1 Demographics of the Respondents

We cleaned the data after collection. The results show that 168 students answered the pre-test and post-test, of which 103 are female, as shown in Table 38.

*Table 38. The number of students who responded*

School	Group	Student list	Female	Drop out	Not complete	Final	Female
A	1	86	55	1	10	75	50
	2	74	36	10	24	40	23
B	2	53	29	13	12	28	18
	1	51	21	15	11	25	12
<b>Total</b>		<b>264</b>	<b>141</b> <b>(53.41%)</b>	<b>39</b> <b>(14.77%)</b>	<b>57</b> <b>(21.59%)</b>	<b>168</b> <b>(63.69%)</b>	<b>103</b> <b>(39.02%)</b>

Noted: - Group 1: Represents the experimental group

- Group 2: Represents the control group

Table 38 shows that students from two schools and four classes were included in the field survey. A total of 264 students were on the list, of which 141 were female. However, 39 students dropped out, and 57 did not complete the pre-and post-test items. They were removed.

### Reliability of test items:

The reliability of the test items was acceptable: Cronbach's Alpha was higher than 0.7 for all items (pilot test, pre-test, and post-test), indicating that the test items are reliable (Gay et al., 2012). Table 39 shows the reliability of the test items.

Table 39. Reliability of the test items using Cronbach's Alpha

Test	Number of items		No. of respondents	Cronbach's Alpha
	CU	VR		
Pilot test 1	5	5	37	0.79
Pilot test 2	5	5	39	0.90
Actual pre-test	5	5	168	0.79
Actual post-test	5	5	168	0.88

### 4.3.2 Descriptive Statistics of Pre-Test and Post-Test on VR and CU

The results of students' responses to test items for VR and CU covering vector and vector operation contents, in general, can be indicated by their responses obtained after taking the test with a correct answer, along with the scoring rubric. The descriptive statistical analysis is shown in Table 40.

Table 40. The descriptive statistics of students' responses through the pre-test and post-test

Descriptive Statistics							
	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error
pre-test of VR	168	1.00	10.00	4.60	1.95	0.42	0.19
post-test of VR	168	3.00	15.00	10.41	2.45	-0.45	0.19
pre-test of CU	168	0.00	11.00	2.92	2.43	0.59	0.19
post-test of CU	168	0.00	15.00	10.45	3.76	-0.71	0.19
Valid N (listwise)	168						

Note: The full score is 15 points for CU and another 15 points for VR

Table 40 illustrates that the average score of students' abilities to solve vectors and operations on vectors of VR is better than that of CU for the pre-test. In the VR tests, the average score of students who solved the problem was 4.60, while in the

CU tests, the result was 2.92. Here, the ability of students to solve geometry problems related to vector concepts depends on their knowledge of VR. It means that students are able to solve geometry problems based on what they can directly see or visualize, but they struggle to think critically about the process of solving geometry problems when the required information is not immediately visible or obvious. This result means that, before the treatment, students' abilities in VR were poor because they could not create a geometrical drawing to clarify the given problems using picture representation. This can lead students to struggle with grasping mathematical concepts and following correct mathematical procedures in relation to picture representations derived from problems given. It makes them unable to solve the problems correctly. This is referred to as poor CU. This result indicates that students with poor performance in VR can have poor performance in CU, meaning they are interrelated.

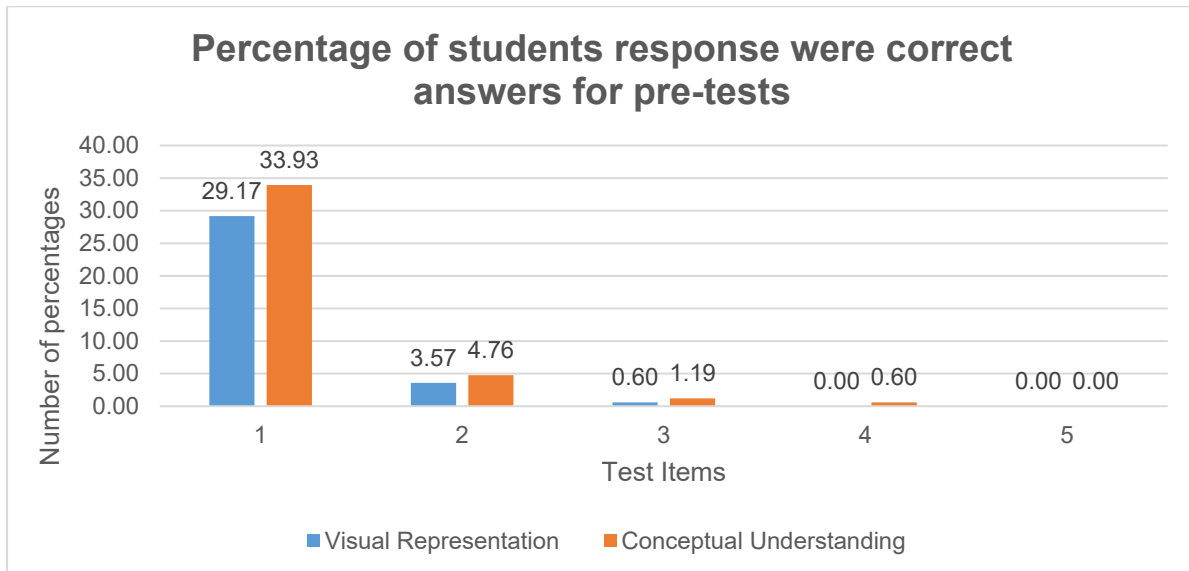
Moreover, the results show a clear improvement in both VR and CU from the pre-test to the post-test. It is important to note that the skewness of the distribution scores of the pre-test and post-test changed from positive to negative, meaning that more students moved from lower to higher performance. The more significant variability in post-test CU score suggests that some students improved more than others in conceptual understanding.

The noticeable difference in pre-test scores between the control and experimental groups may be attributed to several factors, including the selected sample. The students were not randomly chosen, and this significant difference in pre-test scores suggests variations in prior knowledge among them, as they are all in the first grade of upper secondary school (grade 10). According to schoolteachers, "some students in this grade level have transferred from different schools, which may result in varying levels of foundational knowledge from earlier grades, even within the same school."

#### **4.3.3 Students' Improvement in Solving Vector Concepts**

Through the pre-test, students' abilities to answer decrease as the test items progress from 1 to 5, indicating increasing difficulty. Students achieved the highest percentage of correct answers for test item 1 (29.17% for VR and 33.93% for CU). A significant drop in scores is observed for test item 2 (3.57% VR and 4.76% for CU). Test item 3 shows the abilities further decline to 0.60% for VR and 1.19% for CU, while

test item 4 shows that almost no students could answer correctly (0.00% for VR and 0.60% for CU). No students were able to answer test item 5, as both categories show 0.00%. Figure 14 shows the percentage of students who were able to answer correctly in the pre-tests.



*Figure 14. The percentage of students' responses was correct for pre-tests*

Figure 14 shows that, before the intervention, student performance was very low, especially as the test items became more complex from 1 to 5. Both VR and CU declined sharply, suggesting that the students faced difficulties in solving the problems when more complicated and abstract information was added. CU was slightly better than VR in some cases, but the overall ability to answer remained poor. Therefore, this result suggests the need for improved instructional strategies to strengthen students' understanding of geometry.

Moreover, the percentage of students who find it challenging to answer test items in relation to CU is higher than that for VR. The most difficult for the students to answer is referred to as test item 5 in both the pre-test and post-test. It requires students to know how to find the sum of two vectors at the same initial points using a parallelogram. Table 41 shows test item 5 and its expected answer.

Table 41. Test item 5 and its expected answer

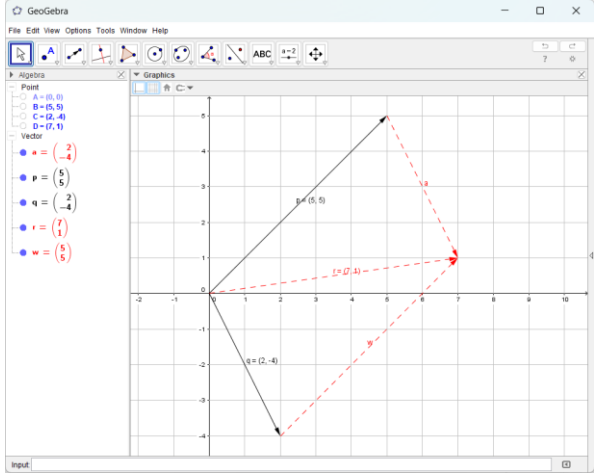
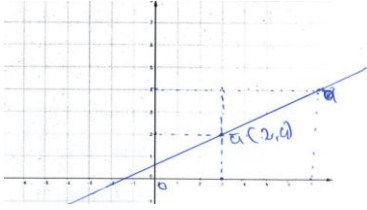
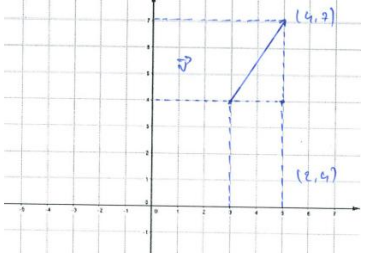
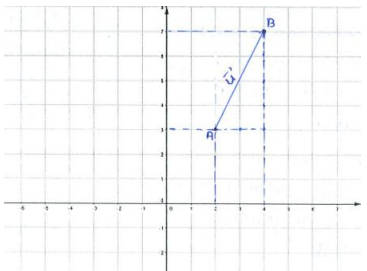
<p>Test item 5. Graph <math>\vec{p} + \vec{q}</math> with the same initial point as <math>\vec{p}</math> and <math>\vec{q}</math>. Explain how you get the answer.</p>	
<p><b>VR</b></p>  <p>By bringing the information from VR, students can explain this problem in CU.</p>	<p><b>CU</b></p> <ul style="list-style-type: none"> <li>- A vector has only one initial point, <b>O</b>.</li> <li>- Draw a parallelogram by graphing the vector.</li> <li>- Draw a diagonal vector from the initial point, <b>O</b>.</li> <li>- Using the parallelogram law, we can write</li> </ul> <p>Let <math>r</math> be the sum of two vectors</p> <p>Then,</p> $r = (5, 5) + (2, -4)$ $= ((5 + 2), (5 - 4))$ $= (7, 1)$ <p>Therefore, <math>r = (7, 1)</math>.</p>

Table 41 shows that students should have the skills necessary for vector operations and understand how to use the parallelogram law by adding two vectors from the same initial points. (1) Students should know the vectors  $\vec{p}$  and  $\vec{q}$  have the same initial point (0,0). (2) Then, draw two vectors from the initial point to the endpoint (0,0). (3) Lastly, using the parallelogram law, they can write  $\vec{p} + \vec{q} = \vec{r}$ . Therefore, by understanding the concepts of VR and CU, students are able to solve this problem.

Students also have difficulty creating geometrical drawings to clarify the problem and grasp mathematical concepts because they have a poor understanding of the foundational concept of vector construction. In this case, students struggled to identify the coordinates of a point, and they could not identify the mathematical expressions along the x-axis and y-axis. These difficulties can cause students to incorrectly set the coordinates of points, as shown in Table 42, which shows an example of students' responses to test item 3.

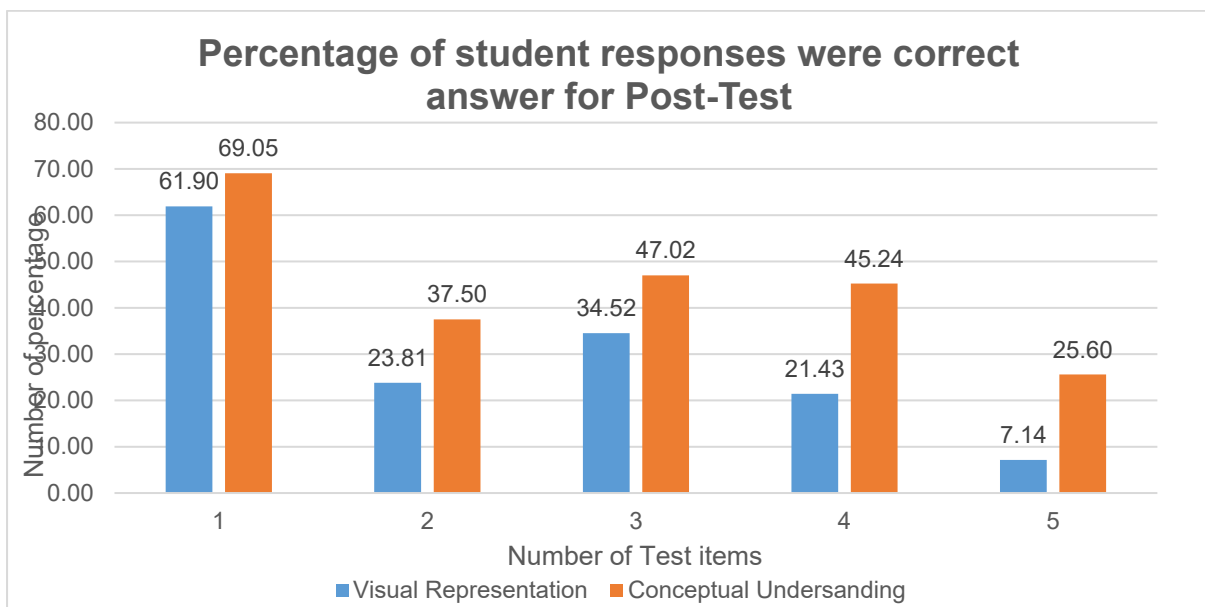
Table 42. An example of students' responses to test item 3

Test Item	VR	CU
3. Draw the vector with an initial point (2,3), then find the terminal point by providing the vector in component form $\vec{u} = (2, 4)$ , and explain how you get such an answer by showing the graph.	(1)	 <p>කොමොන්ට්ස් ටු ඉන්ටර්ප්ලේට් ආරම්භක ස්ථානයේ සිට</p> <p><i>This is because its components are misinterpreting the initial point.</i></p>
	(2)	 <p><math>\vec{u}: B-A = u+A = (2,3)(2,4)</math>  <math>A(2,3) B(4,7)</math></p>
	(3)	 <p>පිටුවේ                      හෙතෙමගේ මගින් ඉටු කිරීමට සිටිනු ලැබේ.                      ඊට පසුව පොදු පරිමාණයෙන් ප්‍රතිචාර දැක්වීම.                      2 ඒකක හරිකම්බල දක්වා දීමට අවශ්‍යය.                      පිටුවේ 4 ඒකක දක්වා දීමට අවශ්‍යය.                      ඉටු කිරීමේදී ප්‍රාය (4,7) /</p> <p><i>I chose this answer because I added two units to the x-axis and four units to the y-axis using the vector given above. Therefore, I obtained (4, 7).</i></p>

Students who had difficulty answering the test items revealed specific characteristics based on their responses. Their solutions were incomplete or contained minor errors; incorrect answers indicated students who provided the answer were completely incorrect, and no response indicated students tended not to understand the problem or did not attempt to answer it. For example, in Table 42, students might struggle to understand what the vector component,  $\vec{u} = (2, 4)$ , means in terms of movement along the x-axis and y-axis, or to accurately plot the initial point (2,3) on the graph. As per the students' responses in Table 42, some common difficulties are apparent, such as (1) misinterpreting the scale or units on the graph, which can lead to incorrect placement; (2) the straight line or arrow drawn from the initial point to the terminal point is not clearly differentiated; (3) inability to clarify the direction and length

of the vector because they must accurately reflect the given components; (4) improper labelling of the initial point, terminal point, and vector on the graph because it is crucial for clarity; (5) unfamiliarity with the coordinate system, including positive and negative direction along the axes. These difficulties can make it challenging for students to grasp the vector representation and plot the points on a graph.

After the intervention, the post-test result shows that the average score for the students' abilities to solve geometry problems with CU is better than that for VR. In CU, the average score of students who solved the problem was 10.45, while in VR, the result was 10.41. However, the students' abilities to solve vector concept problems related to VR and CU are almost similar for each test item because they are interrelated. The post-test results are shown in Figure 15.



*Figure 15. Percentage of students who responded with correct answers on VR and CU*

Figure 15 shows that after the intervention, the percentage of students who answered correctly for CU was higher than that for VR for all items. The highest difference was identified in test item 4, where the difference in the correct responses between CU and VR was 23.81 percentage points. However, in test item 1, students who answered the test item correctly for CU were almost similar to those for VR (7.15 percentage-point difference), indicating that this test item was closely related.

Based on students' calculating procedures for all test items and the result of item analysis, the students encountered difficulties in interpreting VR. These difficulties

included an inability to create geometrical drawings to clarify the given problem and a lack of skills in performing picture representations, including visual, symbolic, and calculation procedures. Similarly, the students faced difficulties in CU owing to an inability to grasp mathematical concepts and perform correct mathematical procedures related to VR.

In analyzing students' responses, we found that the scores indicating their difficulties in solving problems related to vectors and operations on vectors using VR and CU were below the medium level for all test items except for item 1. This suggests that a lack of VR and CU may contribute to poor performance in solving geometry problems because these two cognitive abilities are crucial for effectively grasping and applying geometric concepts. Therefore, students are improved in both VR and CU.

#### **4.3.4 Differentiation between Pre-Test and Post-Test of Students' VR and CU Through Intervention**

As the selection samples were not randomly chosen, we attempted to verify the corresponding data to determine whether it followed a normal distribution for both the pre-test and post-test. In the pre-test, the distribution of scores in the experimental group is slightly skewed to the right, with a mean score of 8.69 and a standard deviation of 3.881. This indicates that most students scored below the mean, and the overall performance before the intervention was relatively low. Similarly, the control group's pre-test scores also show a positively skewed distribution with a mean of 5.79 and a standard deviation of 3.393, suggesting limited prior knowledge in both groups, with the control group scoring slightly lower on average.

After the intervention, the post-test distribution of the experimental group closely approximates a normal distribution. The scores are symmetrically distributed around a much higher mean of 23.05, with a standard deviation of 3.955. This indicates a significant improvement in students' performance, with consistent learning gains and reduced variability. In contrast, the control group's post-test scores are more spread out, with a mean of 17.65 and a standard deviation of 6.754. Although there is a noticeable increase in scores compared to the pre-test, the broader spread suggests greater variability and less consistent improvement among the students. The below figure shows the normal distribution of pre-test and post-test

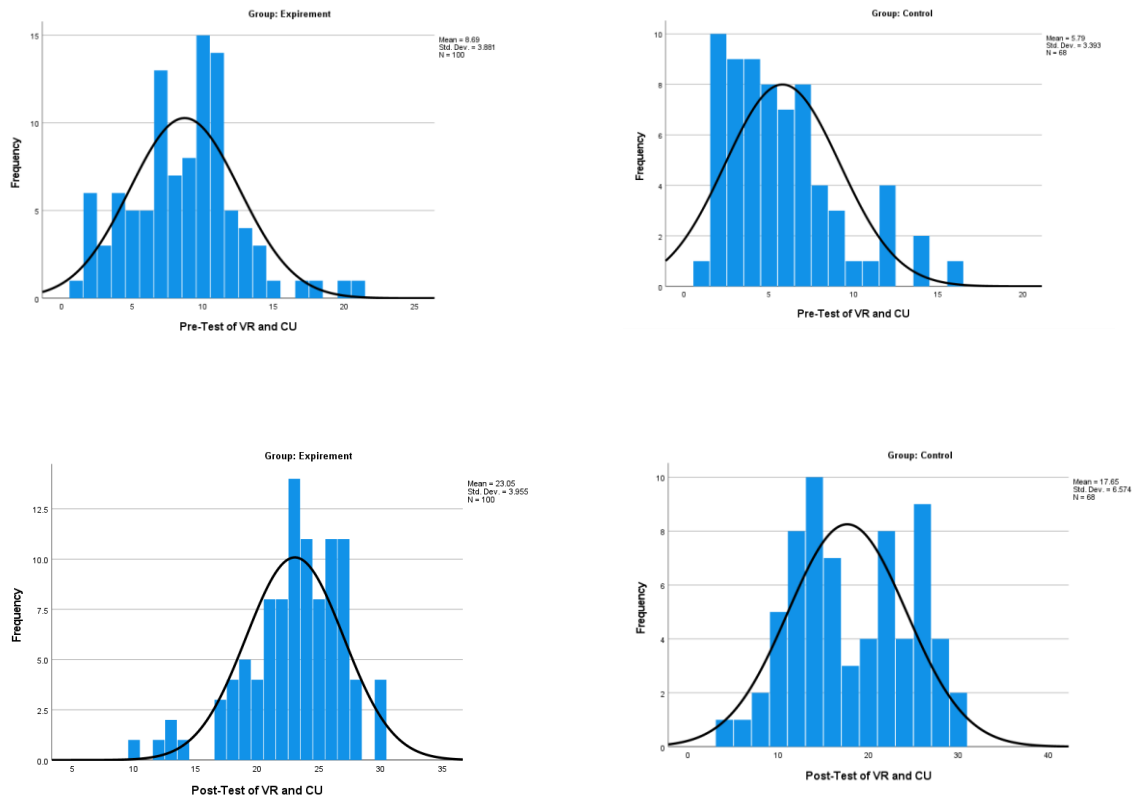


Figure 16. Normal distribution for pre-test and post-test

Figure 16 illustrates that neither set of distributed data follows a normal distribution. In the pre-test, the data is skewed to the left, indicating that most students achieved low scores. By contrast, the post-test data is skewed to the right, suggesting that most students obtained higher scores.

Because the data is not normally distributed, the Mann-Whitney U Test was used to compare the student differences between the experimental and control groups by comparing mean rank. This comparison is based on students' mean rank for CU and VR of contents related to vectors and operations on vectors. The results of this comparison are presented in Table 43.

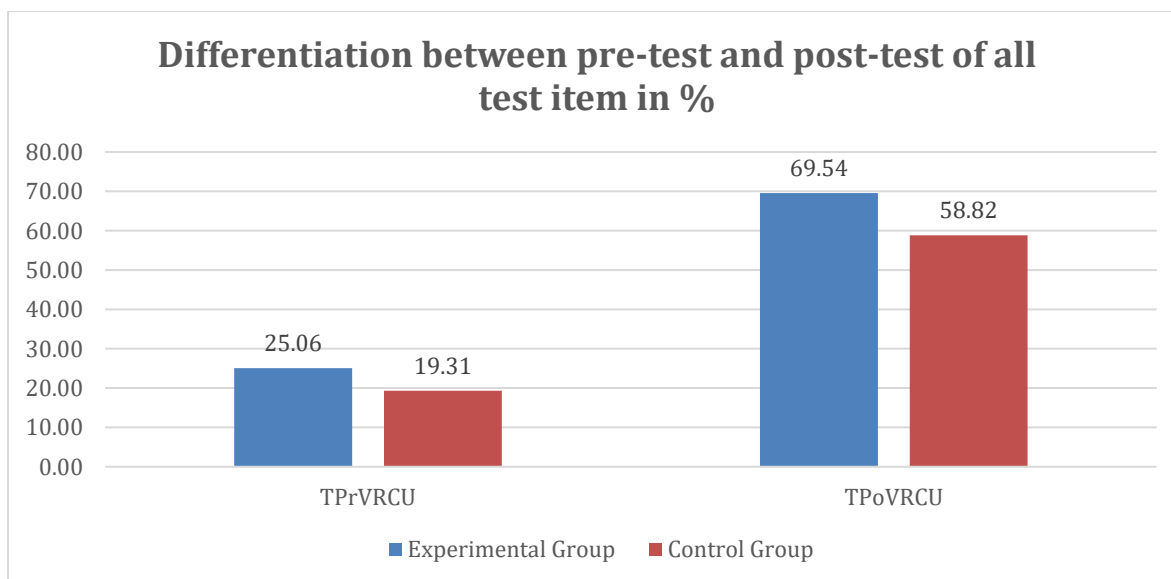
Table 43. The comparison of the improvement in test scores between the pre-test and post-test

		Ranks					
		N	Mean Rank	U	Z	r	Sig
Pre-test	Experimental Group	100	99.61	1889	-4.90	-0.38	p<.001
	Control Group	68	62.28				
	Total	168					
Post-test	Experimental Group	100	100.72	1778.50	-5.25	-0.41	p<.001
	Control Group	68	60.65				
	Total	168					

Table 43 indicates that the experimental group of 100 students had a significantly higher mean rank, 99.61, in the pre-test than the 68 students in the control group, 62.26—U=1889, p<.001, z=-4.90, and r=-0.38. According to Cohen (1988), this represents a medium-to-large effect size. Similarly, a significant difference was found in the mean ranks between the experimental and the control group on the post-test result, where U=1778.50, z=-5.251, p<.001, and r=-0.41, which indicates a medium to large effect size (Cohen, 1988). The results illustrated that the difference between the pre-test and post-test of the VRCU of the control and experimental group shows an increase from the pre-test to the post-test. Moreover, the negative z-scores confirm that the experimental group consistently outperformed the control group in both pre-test and post-test. The large negative value in the post-test, -5.251 versus -4.900, suggests that the gap between the two groups widened after the intervention, meaning the experimental group benefited more than the control group. Furthermore, this result indicates that students need more time to learn about GeoGebra software and use it in an appropriate way to support their learning.

#### 4.3.5 Differentiation of Students' VR and CU by Test Items

Based on the results of the test, the students have improved overall in both the experimental group and the control group. The figure below shows the different improvements of both groups.



*Figure 17. The differentiation between the pre-test and post-test on VR & CU in percentage of the experimental group and the control group*

Figure 17 illustrates the comparison of pre-test and post-test results for VR and CU between an experimental group and a control group. Before the intervention, both groups demonstrated relatively low performance, with the experimental group at 25.06% and the control group at 19.31%. This suggests that before the intervention, students in both groups had limited understanding and ability to solve geometry problems in relation to VR and CU, although the experimental group had a slightly higher starting point. After the instructional intervention, both groups showed improvement; however, the post-test scores reveal a more substantial increase in the experimental group at 69.54% compared to the control group at 58.82%. This significant improvement indicates that the intervention approach applied to the experimental group enhanced students' visual and conceptual skills more effectively. Thus, it is clear evidence of the effectiveness of integrating GeoGebra software in promoting both VR and CU.

#### **4.3.6 Differences Between Control and Experimental Groups Using Normalized Gain Score**

As the scores of the experimental group were higher than those of the control group in the pre-test and post-test very closely, the Normalized Gain Score test was used to differentiate how much the students improved from the pre-test to the post-test. The Normalized Gain Score test is applicable when there is a significant difference

between the experimental group's average post-test value and the control group's post-test value, which is closely achieved. In this way, we try to differentiate the gain from the pre-test to the post-test. The result of the Normalized Gain Score test is shown in Table 44.

Table 44. Normalized Gain Score Percentage (%)

<b>Descriptive</b>							
<b>Experimental group</b>		<b>Statistic</b>	<b>Std. Error</b>	<b>Control group</b>		<b>Statistic</b>	<b>Std. Error</b>
Mean		67.34	1.66	Mean		50.65	2.92
95% Confidence Interval for Mean	Lower Bound	64.05		95% Confidence Interval for Mean	Lower Bound	44.82	
	Upper Bound	70.63			Upper Bound	56.48	
5% Trimmed Mean		67.86		5% Trimmed Mean		50.40	
Median		67.93		Median		46.43	
Variance		275.57		Variance		580.02	
Std. Deviation		16.60		Std. Deviation		24.08	
Minimum		15.00		Minimum		0.00	
Maximum		100.00		Maximum		100.00	
Range		85.00		Range		100.00	
Interquartile Range		23.92		Interquartile Range		41.58	
Skewness		-0.53	0.24	Skewness		0.21	0.29
Kurtosis		0.59	0.48	Kurtosis		-0.95	0.57

Table 44 shows that the average Normalized Gain Score for the experimental group is 67.34%, which is categorized as effective enough, with a minimum score of 15% and a maximum of 100%. Meanwhile, the average Normalized Gain Score for the control group is 50.65%, which is categorized as less effective, with a minimum score of 0% and a maximum of 100% (Hake, 1998).

Therefore, it can be concluded that the use of GeoGebra is effective enough in improving students' VR and CU to overcome learning difficulties in solving geometry problems, particularly those related to vectors and operations on vectors.

#### **4.3.7 Relationship Between VR and CU**

The Spearman's rho correlation analysis is employed to confirm the correlation coefficients between items related to VR and CU for both the experimental and control groups. Each group includes five items measuring VR (PrVRQ1 to PrVRQ5) and five items measuring CU (PrCUQ1 to PrCUQ5). The correlation coefficients indicate the strength and direction of relationships between these items, while the significance values (p-values) show whether these relationships are statistically meaningful. Table 45 presents that in the experimental group, many correlations between VR and CU items are statistically significant at either the 0.05 or 0.01 levels. Notably, strong positive correlations were found between PvVR01 and PrCU01 ( $\rho = .490$ ), PvVR02 and PrCU02 ( $\rho = .635$ ), PvVR04 and PrCU04 ( $\rho = .666$ ), and PvVR05 and PrCU04 ( $\rho = .538$ ), all significant at the 0.01 level. These results suggest that students who scored well on specific visual representation tasks also tended to score well on related conceptual understanding tasks. This pattern of results indicates a strong connection between VR and CU in the experimental group, implying that the intervention helped students develop a coherent understanding of both aspects.

In contrast, the control group also shows some significant correlations, but generally weaker and less consistent than those in the experimental group. For example, PrVRQ1 is moderately correlated with PrCUQ1 ( $\rho = .534$ ) and PrCUQ2 ( $\rho = .562$ ), while PrVRQ2 is correlated with PrCUQ2 ( $\rho = .504$ ), all at the 0.01 level. However, the overall pattern suggests less integration between VR and CU in the control group. Some correlations are also insignificant or very weak, indicating that without the intervention, students' development in visual representation and conceptual understanding may be less connected. The table below shows the relationship between VR and CU.

Table 45. The correlation between VR and CU for the pre-test.

			Correlations											
Group			PrVRQ1	PrCUQ1	PrVRQ2	PrCUQ2	PrVRQ3	PrCUQ3	PrVRQ4	PrCUQ4	PrVRQ5	PrCUQ5		
Spearman's rho	Expirement	PrVRQ1	Correlation Coefficient	1.000	.490**	.447**	.414**	.184	.096	.281**	.256*	.167	.127	
			Sig. (2-tailed)	.	<.001	<.001	<.001	.066	.342	.005	.010	.096	.208	
			N	100	100	100	100	100	100	100	100	100	100	
		PrCUQ1	Correlation Coefficient	.490**	1.000	.323**	.520**	.022	.055	.025	.130	-.027	-.020	
			Sig. (2-tailed)	<.001	.	.001	<.001	.832	.585	.804	.198	.791	.846	
			N	100	100	100	100	100	100	100	100	100	100	
		PrVRQ2	Correlation Coefficient	.447**	.323**	1.000	.635**	.065	.117	.195	.255*	.209*	.099	
			Sig. (2-tailed)	<.001	.001	.	<.001	.519	.247	.052	.011	.037	.329	
			N	100	100	100	100	100	100	100	100	100	100	
		PrCUQ2	Correlation Coefficient	.414**	.520**	.635**	1.000	.121	.300**	.082	.200*	.086	.096	
			Sig. (2-tailed)	<.001	<.001	<.001	.	.230	.002	.420	.047	.394	.343	
			N	100	100	100	100	100	100	100	100	100	100	
		PrVRQ3	Correlation Coefficient	.184	.022	.065	.121	1.000	.511**	.504**	.412**	.396**	.269**	
			Sig. (2-tailed)	.066	.832	.519	.230	.	<.001	<.001	<.001	<.001	.007	
			N	100	100	100	100	100	100	100	100	100	100	
	PrCUQ3	Correlation Coefficient	.096	.055	.117	.300**	.511**	1.000	.295**	.473**	.246*	.543**		
		Sig. (2-tailed)	.342	.585	.247	.002	<.001	.	.003	<.001	.014	<.001		
		N	100	100	100	100	100	100	100	100	100	100		
	PrVRQ4	Correlation Coefficient	.281**	.025	.195	.082	.504**	.295**	1.000	.666**	.598**	.410**		
		Sig. (2-tailed)	.005	.804	.052	.420	<.001	.003	.	<.001	<.001	<.001		
		N	100	100	100	100	100	100	100	100	100	100		
	PrCUQ4	Correlation Coefficient	.256*	.130	.255*	.200*	.412**	.473**	.666**	1.000	.538**	.537**		
		Sig. (2-tailed)	.010	.198	.011	.047	<.001	<.001	<.001	.	<.001	<.001		
		N	100	100	100	100	100	100	100	100	100	100		
	PrVRQ5	Correlation Coefficient	.167	-.027	.209*	.086	.396**	.246*	.598**	.538**	1.000	.618**		
		Sig. (2-tailed)	.096	.791	.037	.394	<.001	.014	<.001	<.001	.	<.001		
		N	100	100	100	100	100	100	100	100	100	100		
	PrCUQ5	Correlation Coefficient	.127	-.020	.099	.096	.269**	.543**	.410**	.537**	.618**	1.000		
		Sig. (2-tailed)	.208	.846	.329	.343	.007	<.001	<.001	<.001	<.001	.		
		N	100	100	100	100	100	100	100	100	100	100		
	Control	PrVRQ1	PrVRQ1	Correlation Coefficient	1.000	.534**	.397**	.562**	.173	-.068	.324**	.212	.100	.171
				Sig. (2-tailed)	.	<.001	<.001	<.001	.157	.580	.007	.083	.416	.164
				N	68	68	68	68	68	68	68	68	68	68
			PrCUQ1	Correlation Coefficient	.534**	1.000	.423**	.514**	.098	.212	.160	.413**	.209	.443**
				Sig. (2-tailed)	<.001	.	<.001	<.001	.425	.082	.193	<.001	.088	<.001
				N	68	68	68	68	68	68	68	68	68	68
			PrVRQ2	Correlation Coefficient	.397**	.423**	1.000	.504**	.243*	.000	.496**	.376**	.444**	.200
				Sig. (2-tailed)	<.001	<.001	.	<.001	.046	1.000	<.001	.002	<.001	.103
				N	68	68	68	68	68	68	68	68	68	68
			PrCUQ2	Correlation Coefficient	.562**	.514**	.504**	1.000	.275*	.243*	.188	.363**	.221	.337**
				Sig. (2-tailed)	<.001	<.001	<.001	.	.023	.046	.125	.002	.070	.005
				N	68	68	68	68	68	68	68	68	68	68
			PrVRQ3	Correlation Coefficient	.173	.098	.243*	.275*	1.000	.319**	.524**	.344**	.203	.125
				Sig. (2-tailed)	.157	.425	.046	.023	.	.008	<.001	.004	.097	.308
				N	68	68	68	68	68	68	68	68	68	68
PrCUQ3		Correlation Coefficient	-.068	.212	.000	.243*	.319**	1.000	.222	.483**	.259*	.372**		
		Sig. (2-tailed)	.580	.082	1.000	.046	.008	.	.068	<.001	.033	.002		
		N	68	68	68	68	68	68	68	68	68	68		
PrVRQ4		Correlation Coefficient	.324**	.160	.496**	.188	.524**	.222	1.000	.450**	.533**	.220		
		Sig. (2-tailed)	.007	.193	<.001	.125	<.001	.068	.	<.001	<.001	.072		
		N	68	68	68	68	68	68	68	68	68	68		
PrCUQ4		Correlation Coefficient	.212	.413**	.376**	.363**	.344**	.483**	.450**	1.000	.494**	.627**		
		Sig. (2-tailed)	.083	<.001	.002	.002	.004	<.001	<.001	.	<.001	<.001		
		N	68	68	68	68	68	68	68	68	68	68		
PrVRQ5		Correlation Coefficient	.100	.209	.444**	.221	.203	.259*	.533**	.494**	1.000	.459**		
		Sig. (2-tailed)	.416	.088	<.001	.070	.097	.033	<.001	<.001	.	<.001		
		N	68	68	68	68	68	68	68	68	68	68		
PrCUQ5		Correlation Coefficient	.171	.443**	.200	.337**	.125	.372**	.220	.627**	.459**	1.000		
		Sig. (2-tailed)	.164	<.001	.103	.005	.308	.002	.072	<.001	<.001	.		
		N	68	68	68	68	68	68	68	68	68	68		

\*\* . Correlation is significant at the 0.01 level (2-tailed).

\* . Correlation is significant at the 0.05 level (2-tailed).

Thus, the experimental group exhibits stronger and more widespread significant correlations between VR and CU items than the control group. This suggests that the instructional intervention effectively enhanced the relationship between these two dimensions of geometry learning. Based on these results, we can conclude that there is a meaningful and reliable connection between the two variables, which may have implications for understanding how these factors interact in the context of this study.

#### 4.4 STUDENT INTERVIEW RESULTS

##### 4.4.1 Selected Criteria for Interview

The students were selected from the total student population to conduct interviews across various levels of student achievement scores on the pre-test of VR and CU. The table below shows the total distribution score of the pre-test of VR and CU.

Table 46. Distribution score of the VR and CU pre-test

<b>Total distribution score of the pre-test of VR and CU</b>					
	Score	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	2	1.19	1.19	1.19
	2	16	9.52	9.52	10.71
	3	12	7.14	7.14	17.86
	4	15	8.93	8.93	26.79
	5	13	7.74	7.74	34.52
	6	12	7.14	7.14	41.67
	7	21	12.50	12.50	54.17
	8	11	6.55	6.55	60.71
	9	11	6.55	6.55	67.26
	10	16	9.52	9.52	76.79
	11	15	8.93	8.93	85.71
	12	9	5.36	5.36	91.07
	13	4	2.38	2.38	93.45
	14	5	2.98	2.98	96.43

15	1	0.60	0.60	97.02
16	1	0.60	0.60	97.62
17	1	0.60	0.60	98.21
18	1	0.60	0.60	98.81
20	1	0.60	0.60	99.40
21	1	0.60	0.60	100.00
Total	168	100.00	100.00	

---

Note: Full score is 30 points.

Table 46 presents the distribution of students' total scores on the pre-test measuring VR and CU of 168 students. The frequencies of the total achievement scores ranged from 1 to 21, indicating varying levels of students' prior knowledge before any instructional intervention. The most frequently occurring score is 7, achieved by 21 students (12.50%).

The majority of students (approximately 85.71%) scored between 1 and 11, suggesting that most students demonstrated low to moderate understanding of VR and CU concepts at the pre-test. Only a small proportion of students (14.29%) scored above 11, indicating a relatively limited number of students with strong prior understanding.

The cumulative percentage gradually increases, reaching 50% by a score of 7 and approximately 85% by a score of 11. This trend reflects a positively skewed distribution, where lower scores are more common than higher scores. The data indicate that most students entered the study with insufficient levels of VR and CU. This justifies the need for targeted instructional strategies or interventions, such as the use of GeoGebra software to support and enhance students' VR and CU skills.

There are different levels for each class, and the scores are categorized based on the pre-test results of each class because we want to see the improvement of both groups from the pre-test to the post-test through the intervention integrating GeoGebra software into teaching and learning mathematics, specifically in geometry and the students' improvement through traditional teaching in schools.. All the distribution scores have been classified into three levels: low, medium, and high. Students who scored between 1 and 7 were categorized as the low-achievement group, those who scored between 8 and 15 were classified as the medium group, and those who scored

between 16 and 21 were classified as the high-achievement group. Among them, we select two from each classified level, which means two from the experimental group and two from the control group. For the experimental group, we selected two students at the lowest score of the low level, two students at the lowest score of the medium level, and two students who achieved the highest score of the high level, based on the pre-test results. For the control group, we did the same as the experimental group. The table below shows the distribution of students per class according to the achievement level in each class.

*Table 47. Distribution of students per class according to the achievement level*

<b>School</b>	<b>Class</b>	<b>High</b>	<b>Medium</b>	<b>Low</b>	<b>total</b>
A	1	9	52	14	75
	2	3	16	21	40
B	1	7	8	13	28
	2	4	9	12	25

Table 47 shows that the frequencies of students' achievements at each level exceeded one, which is higher than the targeted criteria for the interview. Among them, only one student from each level in each class was selected for the interview: the student with the lowest score in the low-achievement group, the student with the lowest score in the medium-achievement group, and the student with the highest score in the high-achievement group.

In cases where student frequencies have more than one at the same lowest, medium or highest score within a group, a lucky draw was used to select one student from each category. At the end, twelve students were selected for an interview to supplement the student responses. The number of participants chosen is shown in the table below.

Table 48. Total distribution score of VR and CU through pre-test

Experimental Group			Control Group		
Category	VR	CU	Category	VR	CU
ExH1	10.00	11.00	CoH1	8.00	6.00
ExH2	9.00	8.00	CoH2	8.00	6.00
ExM1	7.00	8.00	CoM1	6.00	1.00
ExM2	5.00	2.00	CoM2	5.00	2.00
ExL1	1.00	0.00	CoL1	2.00	0.00
ExL2	2.00	0.00	CoL2	1.00	0.00

#### 4.4.2 Students' Improvement Change from Pre-test to Post-test of VR and CU

(1) General scores of interviewed students per achievement level of the experimental and the control group.

Based on the distribution data from the twelve students, both groups improved from the pre-test to the post-test. In the experimental group, all students showed clear improvement in both VR and CU from the pre-test to the post-test. For example, a high-achieving student (ExH1) improved from a pre-test score of 10 in VR to 12 in the post-test and from 11 to 15 in CU. Another high achiever (ExH2) increased from 9 to 13 in VR and from 8 to 15 in CU. Similarly, medium achievers (ExM1 and ExM2) showed significant progress. ExM1 improved by 6 points in both VR and CU, while ExM2 demonstrated even larger gains: a 9-point increase in VR and a 12-point increase in CU. Notably, the low-achieving students (ExL1 and ExL2) made substantial gains. ExL1 improved from 1 to 15 in VR and from 0 to 15 in CU, representing dramatic progress. ExL2 also improved meaningfully, gaining 6 points in both VR and CU. Therefore, the general score of the achievement level of the interviewed students in the experimental and control groups, with twelve students, shows that both groups improved from the pre-test to the post-test. However, the experimental group were higher than the control group, particularly among the students who achieved scores at the low level. The table below shows the change in the students' improvement scores by categories: low, medium, and high achievement scores.

Table 49. Students' improvement scores are categorized by pre-test and post-test of VR and CU.

Experimental Group					Control Group				
Category	VR		CU		Category	VR		CU	
	Pre	Pos	Pre	Pos		Pre	Pos	Pre	Pos
ExH1	10.00	12.00	11.00	15.00	CoH1	8.00	14.00	6.00	13.00
ExH2	9.00	13.00	8.00	15.00	CoH2	8.00	12.00	6.00	15.00
ExM1	7.00	13.00	8.00	14.00	CoM1	6.00	14.00	1.00	14.00
ExM2	5.00	14.00	2.00	14.00	CoM2	5.00	14.00	2.00	14.00
ExL1	1.00	15.00	0.00	15.00	CoL1	2.00	6.00	0.00	3.00
ExL2	2.00	8.00	0.00	6.00	CoL2	1.00	7.00	0.00	3.00

**Note:** The full score for VR is 15, and for CU, it is 15. The total score of both VR and CU is 30.

Table 49 also shows the change in students' improvement in the control group. Students also showed improvements, but the degree of progress was generally minor, especially among the low achievers.

High achievers, such as CoH1, improved from 8 to 14 in VR and from 6 to 13 in CU, and CoH2 progressed similarly.

Medium achievers (CoM1 and CoM2) showed strong gains in CU, with CoM1 improving from 1 to 14 and CoM2 from 2 to 14, indicating some effectiveness of traditional instruction.

Low-achieving students in the control group (CoL1 and CoL2) showed much more limited improvement. CoL1 improved only from 2 to 6 in VR and from 0 to 3 in CU, while CoL2 moved from 1 to 7 in VR and from 0 to 3 in CU.

Thus, while both groups improved from pre-test to post-test, the experimental group, particularly the low achievers, demonstrated more substantial gains in both VR and CU. This result suggests that the intervention used with the experimental group was more effective, especially in supporting students who began with lower levels of understanding.

Although there is an impact at both the higher and medium achievement levels, the differences between pre-test and post-test scores are not significant for these groups. The most considerable difference was observed in the lower achievement

group, indicating a greater impact. Therefore, special attention is given to the lower achievement scores to assess students' improvement during the intervention period.

As explained in the methodology section under the research instruments, the focus of the test item development was to assess students' knowledge progression from the basic test (Test Item 1) to the advanced test (Test Item 5). Therefore, Test Item 1 and Test Item 5 were selected for a more in-depth analysis to supplement the students' responses, particularly those in the lower achievement group.

**(2) Item 1 results per achievement level of CU and VR and the experimental and the control group**

The pre-test and post-test scores for test item 1 of VR (VRQ1) and CU (CUQ1) were categorized by high, medium, and low student performance levels in both the experimental and control groups. Each item has a maximum score of 3 points. In the experimental group, the high-achieving students (ExH1 and ExH2) scored the maximum of 3 points in both VRQ1 and CUQ1 on the pre-test and maintained these scores in the post-test, indicating a consistently strong understanding of the vector and operations on vectors. Medium-achieving students (ExM1 and ExM2) also performed well. While ExM1 already had full scores in the pre-test and maintained them, ExM2 showed notable improvement, increasing from 2 to 3 in VRQ1 and from 1 to 3 in CUQ1. The low-achieving students (ExL1 and ExL2) showed substantial progress. ExL1 improved from 1 to 3 in VRQ1 and from 0 to 3 in CUQ1, while ExL2 improved from 1 to 2 in VRQ1 and from 0 to 2 in CUQ1. This result indicates that the intervention used in the experimental group helped raise the performance of lower-achieving students. The table below presents the students' improvement changes by categories through pre-test and post-test VR and CU of test item 1 by the experimental and control groups among twelve students.

*Table 50. Students' improvement changes by categories for test item 1.*

Experimental Group					Control Group				
Category	ToPre-VRQ1	ToPos-VRQ1	ToPre-CUQ1	ToPos-CUQ1	Category	ToPre-VRQ1	ToPos-VRQ1	ToPre-CUQ1	ToPos-CUQ1
ExH1	3.00	3.00	3.00	3.00	CoH1	3.00	3.00	3.00	3.00
ExH2	3.00	3.00	3.00	3.00	CoH2	3.00	3.00	3.00	3.00
ExM1	3.00	3.00	3.00	3.00	CoM1	3.00	3.00	0.00	3.00

ExM2	2.00	3.00	1.00	3.00	CoM2	3.00	3.00	1.00	3.00
ExL1	1.00	3.00	0.00	3.00	CoL1	1.00	2.00	0.00	2.00
ExL2	1.00	2.00	0.00	2.00	CoL2	1.00	3.00	0.00	3.00

**Note:** The full VR score is 3, and the CU score is 3.

In the control group, the high-achieving students (CoH1 and CoH2) also maintained full scores in both VR and CU. Medium-achieving students (CoM1 and CoM2) improved their scores in CUQ1 of CoM1 from 0 to 3 and CoM2 from 1 to 3 while maintaining full scores in VRQ1. CoL1 showed limited progress among the low-achieving students, improving from 1 to 2 in VRQ1 and from 0 to 2 in CUQ1, while CoL2 reached full scores in both VR and CU test items. Both groups demonstrated learning gains, but the experimental group showed more consistent improvement, especially among low-achieving students. This result suggests that the intervention applied to the experimental group was particularly effective in enhancing both visual and conceptual understanding for students at different achievement levels.

Moreover, the development of test item 1 is to assess student knowledge within the context of vectors and operations on vectors, and it is to support the development of students' VR and CU in coordinate geometry. The activity engages students in plotting specific points, such as  $A(3, 2)$ ,  $B(-2, 1)$ ,  $C(4, -1)$ , and  $D(-1, -2)$  on the Cartesian coordinate plane. This test item enables students to translate symbolic representations into spatial forms, thereby fostering meaningful connections between abstract numerical information and concrete geometric positioning.

From the perspective of VR part, the task assesses students' ability to recognize, interpret, and construct visual images of coordinate points. It encourages external representations (e.g., graphing on a coordinate plane) while supporting mental operations as students visualize the relative position of points before plotting them. Such experiences are critical for enhancing spatial reasoning and promoting the interpretation of geometric information in visual form.

In the conceptual understanding part, the task assesses students' internalisation of the Cartesian plane's structure and function, including the roles of the x-axis, y-axis, and origin. Through plotting and explaining their reasoning, students develop a deeper understanding of the properties of the coordinate system and the mathematical logic governing point placement. This task promotes procedural fluency

and an understanding of the relationships between algebraic and geometric. The figure below presents test item 1 used as a pre-test and a post-test.

1: Set four points which are given  $A(3,2)$ ,  
 $B(-2,1)$ ,  $C(4,-1)$  and  $D(-1,-2)$  on the below figure and explain how you get the answer.

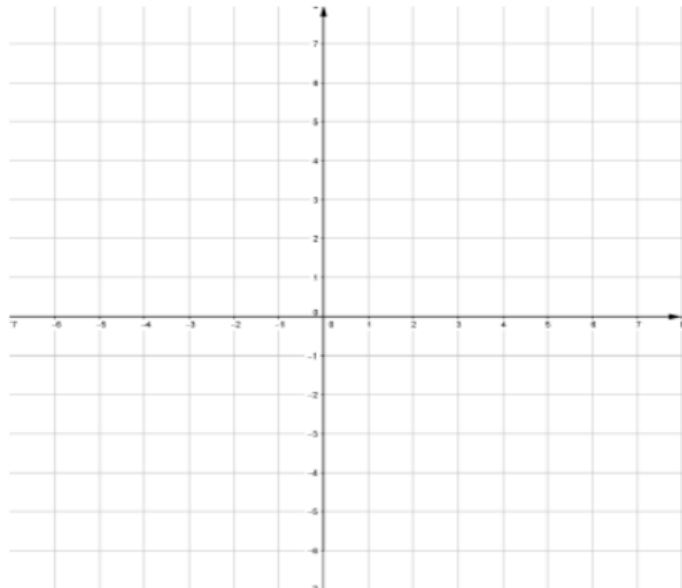


Figure 18. Test item 1

The students' results revealed that students with high achievement scores can solve this test item correctly in both the pre-test and post-test. In terms of VR, they can recognize and interpret information, including visual, symbolic, and verbal representations from the given problem. In terms of CU, they can explain the mathematical concept following the mathematical procedure, providing reasoning in relation to VR. The interview with a focus on VR reveals the following:

**Note:** R: Researcher; ExH1\_2: High achievement students 1 and 2 of the experimental group; CoH1\_2: Low achievement students 1 and 2 of the control group.

The researchers used semi-structured interviews to obtain the data. The interview focused on the students' ability to solve geometric problems, specifically regarding vectors and operations on vectors concerning VR and CU. The aim was to assess the students' ability to create a geometrical drawing and their explanation based on the definitions of VR and CU in this study. The results are presented as follows:

**(3) Item 1 interview protocols per achievement level, the control group and the experimental group**

**[Low-achievement students, CoL1, CoL2, from the control group]**

The students' results from the test are shown in the table below.

*Table 51. Low-achievement students' response to the test item 1 from the control group.*

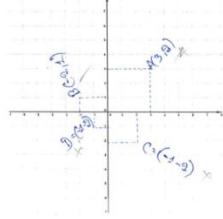
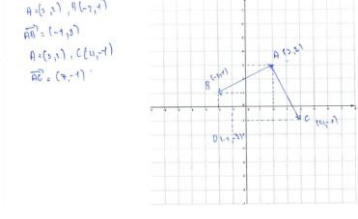
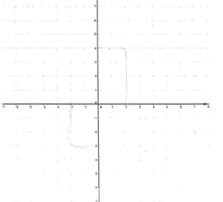

Student	Pre-Test	Post-Test
CoL1	<p>១. គេឱ្យប្រគល់កូអរដោនេ <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ប្រគល់កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ និងគូសរូបវ៉ិចទ័រដែលមានចំនុចដើមនៅកូអរដោនេខ្ទាំងខាងលើ៖</p> 	<p>១. គេឱ្យប្រគល់កូអរដោនេ <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ប្រគល់កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ និងគូសរូបវ៉ិចទ័រដែលមានចំនុចដើមនៅកូអរដោនេខ្ទាំងខាងលើ៖</p> <p> <math>A(3,2)</math>, <math>B(-2,1)</math>  <math>\vec{AB} = (-5, -1)</math>  <math>B(-2,1)</math>, <math>C(4,-1)</math>  <math>\vec{BC} = (6, -2)</math>  <math>D(-1,-2)</math> </p> 
CoL2	<p>១. គេឱ្យប្រគល់កូអរដោនេ <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ប្រគល់កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ និងគូសរូបវ៉ិចទ័រដែលមានចំនុចដើមនៅកូអរដោនេខ្ទាំងខាងលើ៖</p> 	<p>១. គេឱ្យប្រគល់កូអរដោនេ <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ប្រគល់កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ និងគូសរូបវ៉ិចទ័រដែលមានចំនុចដើមនៅកូអរដោនេខ្ទាំងខាងលើ៖</p> <p> <math>A(3,2)</math> កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ  <math>B(-2,1)</math> កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ  <math>C(4,-1)</math> កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ  <math>D(-1,-2)</math> កូអរដោនេចំនុចកណ្តាលនៃចតុកោណ         </p> 

Table 51 presents the responses of low-achievement students from the control group to Test Item 1, comparing their performance between the pre-test and post-test of two students, CoL1 and CoL2. In the pre-test, student CoL1 attempted to plot several points on the coordinate plane; however, the placement of these points appeared inaccurate, and there was no clear indication of vector construction or correct interpretation of vector positions. This suggests a limited understanding of the task. In contrast, the post-test response of CoL1 showed noticeable improvement. The student correctly identified and plotted the points and vectors, such as  $\vec{AB}$ ,  $\vec{BC}$ , and  $\vec{AC}$  and provided written calculations and reasoning. This indicates a better grasp of vector representation and coordinate plotting.

Similarly, Student CoL2 did not provide any meaningful response in the pre-test. Although a coordinate plane was drawn, no vectors or points were plotted, reflecting a lack of understanding or inability to begin the task. However, in the post-

test, CoL2 demonstrated significant progress by plotting points, drawing vectors, and including relevant calculations. This response indicates an improved conceptual understanding, even though the student was part of the control group.

Thus, both students showed improvement from pre-test to post-test, particularly in their ability to represent vectors and points on the coordinate plane. Although they were in the control group and did not receive the intervention, their performance suggests some learning gains through traditional instruction. The interview results are revealed in the table below.

Table 52. The improvement in students' results based on the interview regarding test item 1 for students with low achievement

<b>Aspect</b>	<b>Pre-Test</b>	<b>Post-Test</b>
<b>Visual Representation</b>	<p><b>R. How do you draw this picture?</b></p> <p><i>CoL1. "... I plotted the points followed by the given problem, but I am wrong in plotting the coordinates of point A..."</i></p> <p><i>CoL2. "... I have no idea how to draw this picture, followed by the instructions of the given problem..."</i></p> <p><b>R. Why do you draw this picture in this way?</b></p> <p><i>CoL1. "... I am interpreting the coordinates of points from the given problem..."</i></p> <p><i>CoL2. "... I have no idea..."</i></p> <p><b>R. Why do you have this mistake?</b></p> <p><i>CoL1. "... I confuse the x-axis with the y-axis for point A..."</i></p> <p><i>CoL2. "... I have no idea..."</i></p>	<p><b>R. How do you draw this picture?</b></p> <p><i>CoL1 and CoL2 "... I plotted the coordinates of points followed by the instruction of the given problem..."</i></p> <p><b>R. Why do you draw this picture in this way?</b></p> <p><i>CoL1. "... I confused the interpretation of point A..."</i></p> <p><i>CoL2. "...I use the sign of coordinate and quadrant..."</i></p> <p><b>R. Why do you have this mistake?</b></p> <p><i>CoL1. "... I misinterpret the coordinate of point A..."</i></p>

		<i>CoL2. "... there is a correct picture..."</i>
<b>Conceptual understanding</b>	<b>R. How do you understand this problem?</b> <i>CoL1 and CoL2. "... I cannot understand ..."</i> <b>R. How do you solve this problem and get this result?</b> <i>CoL1 and CoL2. "... I have no idea..."</i>	<b>R. How do you understand this problem?</b> <i>CoL1 and CoL2. "... I read the given problem and use a mental image to think. "</i> <b>R. How do you solve this problem and get this result?</b> <i>CoL1. "... I do not know the procedure for solving this problem..."</i> <i>CoL2. "... I explained it by using the coordinates of points followed by the x-axis and the y-axis..."</i>

Through this interaction, it can be concluded that the responses from the two low-achievement students in the control group reveal noticeable improvement from pre-test to post-test in both visual representation and conceptual understanding, despite the absence of targeted intervention. Initially, both students demonstrated limited understanding, with CoL1 making specific errors in coordinate plotting and CoL2 expressing complete confusion. By the post-test, both students were able to follow instructions more accurately, plot points correctly, and articulate their reasoning more clearly. While CoL1 still struggled with certain concepts, CoL2 showed greater confidence and understanding of coordinate systems. These findings suggest that even without intervention, regular instruction may contribute to gradual improvement in students' geometric thinking, particularly in visual and conceptual aspects.

**[Low-achievement students, ExL1, ExL2, from the experimental group]**

The students' results from the test are shown in the table below, presented to low-achievement students from the experimental group through the test item 1.

Table 53. Low-achievement students' response to the test item 1 from the experimental group

Student	Pre-Test	Post-Test
ExL1	<p>9. គេផ្តល់ចំណុច <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ចូរដាក់ចំណុចទាំងបួននៅលើប្រភាព ឆ្នងក្រាប និងបញ្ជាក់ទំនាក់ទំនងរវាងវ៉ិចទ័រទាំងបួនដោយប្រើប្រាស់អ័ក្សបែបនេះ៖</p>	<p>9. គេផ្តល់ចំណុច <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ចូរដាក់ចំណុចទាំងបួននៅលើប្រភាព ឆ្នងក្រាប និងបញ្ជាក់ទំនាក់ទំនងរវាងវ៉ិចទ័រទាំងបួនដោយប្រើប្រាស់អ័ក្សបែបនេះ៖</p> <p>ចម្លើយ៖</p> <p>ចំណុចទាំងបួនដែលបានផ្តល់ឱ្យគេ គេដាក់នៅលើប្រភាពឆ្នងក្រាបដូចខាងក្រោម៖</p>
ExL2	<p>9. គេផ្តល់ចំណុច <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ចូរដាក់ចំណុចទាំងបួននៅលើប្រភាព ឆ្នងក្រាប និងបញ្ជាក់ទំនាក់ទំនងរវាងវ៉ិចទ័រទាំងបួនដោយប្រើប្រាស់អ័ក្សបែបនេះ៖</p>	<p>9. គេផ្តល់ចំណុច <math>A(3,2)</math>, <math>B(-2,1)</math>, <math>C(4,-1)</math> និង <math>D(-1,-2)</math>។ ចូរដាក់ចំណុចទាំងបួននៅលើប្រភាព ឆ្នងក្រាប និងបញ្ជាក់ទំនាក់ទំនងរវាងវ៉ិចទ័រទាំងបួនដោយប្រើប្រាស់អ័ក្សបែបនេះ៖</p> <p>ដើម្បីដាក់ចំណុចទាំងបួនដែលបានផ្តល់ឱ្យគេ នៅលើប្រភាពឆ្នងក្រាប គេត្រូវដឹងពីតម្លៃ x និង y នៃចំណុចនីមួយៗ។</p>

Table 53 presents the responses of two low-achievement students, ExL1 and ExL2, from the experimental group, comparing their performance on Test Item 1 between the pre-test and post-test.

In the pre-test, ExL1 attempted to plot points on the coordinate plane, but the placement of some points was inaccurate, and the vector relationships were not clearly represented. There was also no written explanation provided. In contrast, the post-test response showed significant improvement. ExL1 correctly plotted the points and vectors and provided a detailed written explanation, indicating enhanced understanding of coordinates and vector construction.

Similarly, ExL2 did not plot any points or provide any explanation in the pre-test, reflecting a lack of understanding. However, in the post-test, the student accurately plotted the required points and vectors and included a written explanation that demonstrated correct reasoning using the x- and y-axes.

Based on the test results, both students showed clear improvement in visual representation and conceptual understanding from pre-test to post-test. Their progress suggests that the instructional intervention used in the experimental group,

particularly the integration of GeoGebra software, was effective in supporting low-achieving students' learning in geometry.

Moreover, through the interview, the students responded as shown in the table below.

*Table 54. The improvement in students' results based on the interview regarding test item 1 for students with low achievement*

<b>Aspect</b>	<b>Pre-Test</b>	<b>Post-Test</b>
<b>Visual Representation</b>	<p><b>R. How do you draw this picture?</b>  <i>ExL1. "... I plot the points followed by the given problem..."</i>  <i>ExL2. "... I plotted the point followed by the given problem, but I did not express the coordinates of the point represented in the picture"</i></p> <p><b>R. Why do you draw this picture in this way?</b>  <i>ExL1 and ExL2. "... I don't know how to draw this picture because I did not understand the given problem concepts..."</i></p> <p><b>R. Why do you have this mistake?</b>  <i>ExL1 and ExL2 "... I have no idea..."</i></p>	<p><b>R. How do you draw this picture?</b>  <i>ExL1 and ExL2. "... I can plot this picture based on the use of the coordinates of points..."</i></p> <p><b>R. Why do you draw this picture in this way?</b>  <i>ExL1. "... I express the points by using a quadrant and the sign of coordinates..."</i>  <i>ExL2. "... I did not pay attention to the coordinates expression..."</i></p> <p><b>R. Why do you have this mistake?</b>  <i>ExL2. "... I just write the symbol only..."</i></p>

<b>Conceptual understanding</b>	<b>R. How do you understand this problem?</b> <i>ExL1 and ExL2. "... I understand the problems, but I don't know how to plot the points followed by the given problem..."</i> <b>R. How do you solve this problem and get this result?</b> <i>ExL1 and ExL2. "... I have no idea how to explain..."</i>	<b>R. How do you understand this problem?</b> <i>ExL1 and ExL2. "... I can explain this process by remembering the way that the teacher usually explains in the class..."</i> <b>R. How do you solve this problem and get this result?</b> <i>ExL1 and ExL2. "... I can explain them, followed by the quadrant and the sign of coordinates..."</i>
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Based on the interaction, the responses of ExL1 and ExL2 from the experimental group show clear improvement in both visual representation and conceptual understanding from pre-test to post-test. Initially, both students struggled with interpreting the problem and plotting points correctly. After the intervention, they demonstrated better use of coordinates, awareness of quadrants, and the ability to explain their reasoning based on classroom instruction. This suggests that the intervention by integrating GeoGebra software into teaching and learning geometry effectively supported their development in understanding and visualizing geometric concepts.

**(4) Item 5 results per achievement level of CU and VR and the experimental and the control group.**

The pre-test and post-test scores of students in both the experimental and control groups for one visual representation item (VRQ5) and one conceptual understanding item (CUQ5) each scored a maximum of 3 points. The students are categorized by their performance levels: high, medium, and low. In the experimental group, most students started with scores of 0 or 1 in both VRQ5 and CUQ5 during the pre-test. After the intervention, all students showed improvement in VR, and nearly all

improved in CU. For example, ExH1 increased from 0 to 2 in VRQ5 and from 0 to 3 in CUQ5, and ExL1 improved from 0 to 3 in both test items. While ExL2 showed limited progress in VRQ5 (0 to 1), they did not improve in CUQ5. Thus, the majority of students in the experimental group demonstrated significant gains. The table below shows the data on the change in students' improvement throughout the pre-test and the post-test of test item 5 for the experimental and control groups.

Table 55. students' improvement throughout the pre-test and the post-test of test item 5.

Experimental Group					Control Group				
Category	ToPre-VRQ5	ToPos-VRQ5	ToPre-CUQ5	ToPos-CUQ5	Category	ToPre-VRQ5	ToPos-VRQ5	ToPre-CUQ5	ToPos-CUQ5
ExH1	0.00	2.00	0.00	3.00	CoH1	1.00	2.00	1.00	2.00
ExH2	1.00	2.00	2.00	3.00	CoH2	1.00	2.00	1.00	3.00
ExM1	0.00	2.00	0.00	3.00	CoM1	0.00	3.00	0.00	3.00
ExM2	0.00	2.00	0.00	2.00	CoM2	0.00	2.00	0.00	2.00
ExL1	0.00	3.00	0.00	3.00	CoL1	0.00	1.00	0.00	0.00
ExL2	0.00	1.00	0.00	0.00	CoL2	0.00	0.00	0.00	0.00

In the control group, high and medium achievers also improved. For instance, CoH2 increased from 1 to 2 in VRQ5 and from 1 to 3 in CUQ5, and CoM1 improved from 0 to 3 in both items. However, the low-achieving students showed much less progress. CoL1 increased only from 0 to 1 in VRQ5 and remained at 0 in CUQ5, while CoL2 showed no improvement at all, remaining at 0 for both items. Furthermore, although both groups showed some level of improvement, the experimental group exhibited more consistent and greater gains, especially among lower-achieving students. This suggests that the instructional intervention used in the experimental group was more effective in enhancing both visual representation and conceptual understanding.

The purpose of test item 5 is to assess students' understanding of vector addition through VR and CU. From a VR perspective, the test item guides students to construct vectors  $\vec{p} + \vec{q}$  starting from the same initial point on the coordinate plane. Then, students are expected to visually complete a parallelogram using these two vectors as adjacent sides. The task requires them to draw the diagonal of the parallelogram, which visually represents the sum of two vector  $\vec{p} + \vec{q}$ . This process

strengthens students' ability to recognize geometric patterns, interpret vector directions and magnitudes, and translate mathematical concepts into graphical forms.

From a conceptual understanding perspective, the test item deepens students' reasoning about vector operations. It moves beyond procedural calculation by demonstrating that vector addition involves forming a geometric shape (parallelogram), where the diagonal expresses the total effect of the two vectors. Students learn that vector addition is not arbitrary but follows a logical spatial rule rooted in geometry. By applying the parallelogram law, students gain insight into the meaning and structure of vector addition, supporting their ability to explain and justify the result visually and conceptually. The figure below shows test item 5.

5. Graph  $\vec{p} + \vec{q}$  with the same initial point as  $\vec{p}$  and  $\vec{q}$ . Explain how you got the answer.

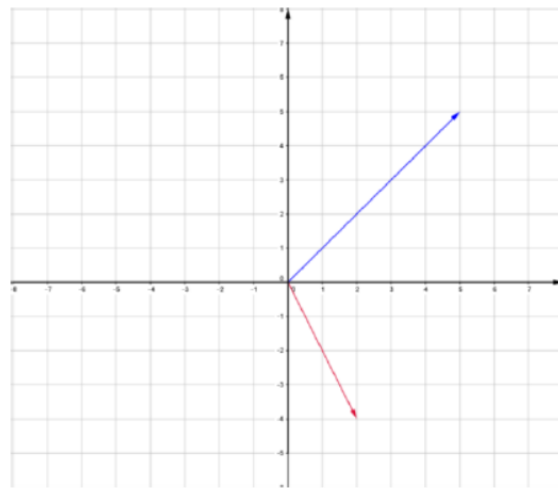


Figure 19. The test items 5

**(5) Item 5 interview protocols per achievement level, the control group and the experimental group**

**[Low-achievement students, CoL1, CoL2, from the control group]**

The improvement change from the test item 5 is shown in the table below.

Table 56. Low-achievement students' response to the test item 5 from the control group.

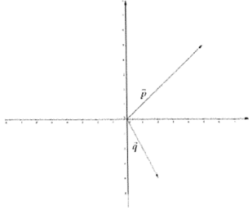
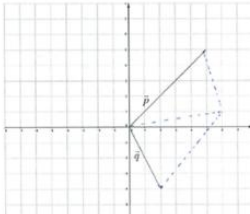
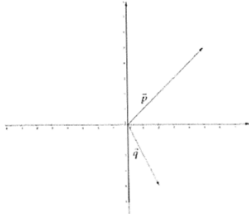
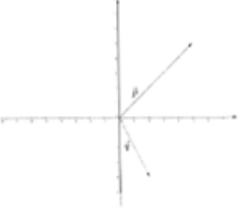
Student	Pre-Test	Post-Test
CoL1	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p} + \vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរពន្យល់ខ្លី ហេតុអ្វីដែលប្រូមេត្រីសព័សតម្លើយសមរម្យ?</p> 	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p} + \vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរពន្យល់ខ្លី ហេតុអ្វីដែលប្រូមេត្រីសព័សតម្លើយសមរម្យ?</p> 
CoL2	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p} + \vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរពន្យល់ខ្លី ហេតុអ្វីដែលប្រូមេត្រីសព័សតម្លើយសមរម្យ?</p> 	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p} + \vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរពន្យល់ខ្លី ហេតុអ្វីដែលប្រូមេត្រីសព័សតម្លើយសមរម្យ?</p> 

Table 56 presents the pre-test and post-test responses of two low-achieving students, CoL1 and CoL2, from the control group on Test Item 5, which focused on vectors and operations on vectors.

In the pre-test, both students showed limited understanding. CoL1 drew basic vectors without completing any meaningful shape, and CoL2 also produced a simple vector diagram with no clear structure. However, in the post-test, CoL1 demonstrated noticeable improvement by accurately constructing a parallelogram, indicating an improved understanding of vector addition and geometric representation. In contrast, CoL2's post-test response showed no progress, as the drawing remained nearly the same with no further development. Thus, the results indicate that while some students in the control group may improve through traditional instruction, others show minimal progress without additional support.

Moreover, the interview results are presented in the table below.

Table 57. The improvement in students' results based on the interview regarding test item 1 for students with low achievement

Aspect	Pre-Test	Post-Test
Visual Representation	<p><b>R. How can you draw these pictures?</b></p> <p><i>CoL1 and CoL2. "...I do not know how to draw this picture, ...I have no idea how to draw vector <math>p</math> + vector <math>q</math>".</i></p>	<p><b>R. How can you draw these pictures?</b></p> <p><i>CoL1. "...I drew this picture because I remember that the sum of two vectors at the same points is an equal diagonal parallelogram... "</i></p> <p><i>CoL2. "... I did not remember what the teacher explained about this theorem..."</i></p>
	<p><b>R. Why don't you draw this picture?</b></p> <p><i>CoL1. "... I don't have any idea how to draw..."</i></p> <p><i>CoL2. "... I have no idea..."</i></p>	<p><b>R. Why do you draw pictures in these ways?</b></p> <p><i>CoL1. "... I drew this picture by drawing a parallelogram..."</i></p> <p><i>CoL2. "... I have no idea..."</i></p>
	<p><b>R. Do you have any idea how to draw this picture?</b></p> <p><i>CoL1 and CoL2. "... No. I don't have..."</i></p>	<p><b>R. Why do you have these mistakes?</b></p> <p><i>CoL1 and CoL2. "...I usually do like this..."</i></p>
Conceptual understanding	<p><b>R. How do you understand this problem?</b></p> <p><i>CoL1 and CoL2. "...I understand the problem, but I can not explain ..."</i></p>	<p><b>R. How do you understand this problem?</b></p> <p><i>CoL1 and CoL2. "...I understand the problem, but I can not explain ..."</i></p>
	<p><b>R. How do you solve these problems and get these results?</b></p> <p><i>CoL1 and CoL2. "...I don't know how to explain..."</i></p>	<p><b>R. How do you solve these problems and get these results?</b></p> <p><i>CoL1 and CoL2. "...I don't know how to explain..."</i></p>

Through the interaction, the responses from CoL1 and CoL2 show that both students in the control group initially struggled to represent and explain vector addition. While CoL1 demonstrated some improvement in visual representation by recalling and applying the parallelogram method in the post-test, CoL2 showed little to no progress. However, both students continued to struggle with explaining their reasoning, indicating limited conceptual understanding. These results suggest that traditional instruction alone may not be sufficient for low-achieving students to grasp abstract vector concepts without additional support or intervention.

**[Low-achievement students, ExL1, ExL2, from the experimental group]**

The students' results from test item 5 from the experimental group are shown in the table below.

*Table 58. Low-achievement students' response to the test item 5 from the experimental group*

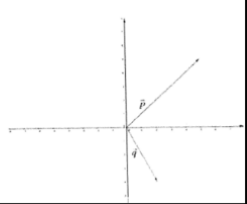
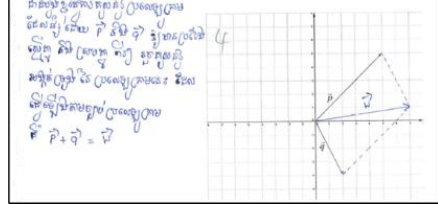
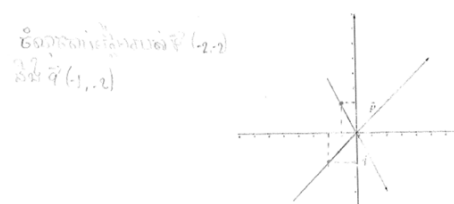
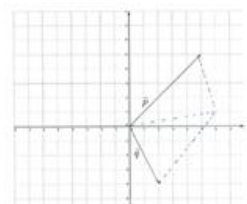
Student	Pre-Test	Post-Test
ExL1	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p}</math> និង <math>\vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរគូសរូបរាងប៉ារ៉ាឡេក្រាមដើម្បីសរុបវ៉ិចទ័រទាំងពីរ។</p> 	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p}</math> និង <math>\vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរគូសរូបរាងប៉ារ៉ាឡេក្រាមដើម្បីសរុបវ៉ិចទ័រទាំងពីរ។</p> <p>ដើម្បីសរុបវ៉ិចទ័រ <math>\vec{p}</math> និង <math>\vec{q}</math> យើងប្រើវិធីសាស្ត្រប៉ារ៉ាឡេក្រាម។ យើងគូសបន្ទាត់ប្រដាប់ពីចំណុចចុងរបស់ <math>\vec{p}</math> ទៅចំណុចចុងរបស់ <math>\vec{q}</math> និងច្របាច់បន្ទាត់ទាំងពីរ។ វ៉ិចទ័រសរុបគឺជាវ៉ិចទ័រ <math>\vec{p} + \vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដល់ចំណុចចុងរបស់ប៉ារ៉ាឡេក្រាម។</p> 
ExL2	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p}</math> និង <math>\vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរគូសរូបរាងប៉ារ៉ាឡេក្រាមដើម្បីសរុបវ៉ិចទ័រទាំងពីរ។</p> <p>ចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> គឺ <math>(-2, 2)</math>          ចំណុចចាប់ផ្តើមរបស់ <math>\vec{q}</math> គឺ <math>(-1, 1)</math></p> 	<p>៥. ចូរគូសវ៉ិចទ័រ <math>\vec{p}</math> និង <math>\vec{q}</math> ដែលចេញពីចំណុចចាប់ផ្តើមរបស់ <math>\vec{p}</math> និង <math>\vec{q}</math> ដូចរូបខាងក្រោម។ ចូរគូសរូបរាងប៉ារ៉ាឡេក្រាមដើម្បីសរុបវ៉ិចទ័រទាំងពីរ។</p> 

Table 58 shows the improvement of two low-achieving students, ExL1 and ExL2, from the experimental group in solving Test Item 5. In the pre-test, both students demonstrated limited understanding, drawing only basic vectors without accurately representing vector addition. In the post-test, both students showed clear progress by correctly constructing a parallelogram to represent the sum of two vectors. ExL1 also provided a written explanation, showing stronger conceptual understanding. This

suggests that the instructional intervention effectively supported their development in both visual representation and conceptual understanding of vector addition.

The results of the interview revealed as table below.

Table 59. The improvement in students' results based on the interview regarding test item 5 for students with low achievement in the experimental group.

Aspect	Pre-Test	Post-Test
Visual Representation	<p><b>R. How do you draw this picture?</b>  <i>ExL1. "... I have no idea..."</i>  <i>ExL2. "... my understanding is like what I am confusing..."</i></p> <p><b>R. Why don't you draw this picture in this way?</b>  <i>ExL1. "Ah... I have no idea about that..."</i>  <i>ExL2. "...I also don't know how to I draw this picture..."</i></p> <p><b>R. Why don't you try to draw this picture?</b>  <i>ExL1. "... I have no idea..."</i>  <i>ExL2. "... I have no idea how to solve this problem..."</i></p>	<p><b>R. How can you draw this picture?</b>  <i>ExL1 and ExL2. "...in applying the GeoGebra lesson, I can draw this picture by the given problem of the sum of vector <math>p \vec{+} q \vec{}</math>. And I apply the parallelogram law..."</i></p> <p><b>R. How do you draw this picture in this way?</b>  <i>ExL1 and ExL2. "... parallel line along <math>p \vec{}</math> and <math>q \vec{}</math>, then by the law of a parallelogram, a diagonal of the parallelogram is the sum of two vectors at the same initial points..."</i></p> <p><b>R. Why do you make this mistake?</b>  <i>ExL1. "... I forgot to write the coordinates of a point. I thought that there was no need..."</i>  <i>ExL2. "... I suppose that this is the final result..."</i></p>

<p><b>Conceptual understanding</b></p>	<p><b>R. How do you understand this problem?</b>  <b>ExL1 and ExL2.</b> "... By reading the problems, I understand the idea of the problems..."</p> <p><b>R. How do you solve this problem and get this result?</b>  <b>ExL1 and ExL2.</b> "... I have no idea how to solve these problems and how to solve that problem..."</p>	<p><b>R. How do you understand this problem?</b>  <b>ExL1 and ExL2.</b> "... After learning and using GeoGebra software, I can identify and explain the coordinates of points from the pictures because it provided an algebraic explanation..."</p> <p><b>R. How do you solve this problem and get this result?</b>  <b>ExL1.</b> "...I explained, followed by the picture representation and my understanding after learning with GeoGebra. The results are the same..."  <b>ExL2.</b> "...I don't know how to explain the result..."</p>
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Through the coverstation, we can conclude that both ExL1 and ExL2 demonstrated significant improvement in visual representation and conceptual understanding of vector addition after using GeoGebra. While they initially had no idea how to approach the problem, the post-test responses showed they were able to apply the parallelogram law and use coordinates more effectively. GeoGebra supported their understanding by providing visual and algebraic guidance. Although minor mistakes remained, such as missing coordinates or difficulty explaining the result, the overall progress highlights the effectiveness of the intervention in supporting low-achieving students.

### 4.4.3 Summary of Students' Interview Results

Table 60 shows the summaries of the interviews with students from both the experimental and control groups regarding the intervention lessons for solving problems related to vectors and operations on vectors by test items.

Table 60. Summary of the interview results

Participants ' group	Conceptual Understanding	Visual Representation
Low	<ul style="list-style-type: none"> <li>- Students are unable to explain the mathematical procedures they use by referring to underlying mathematical concepts, such as properties of shapes or calculation steps (properties)</li> <li>- Students are unable to provide logical reasoning or justification for the relationships between geometric elements, indicating a lack of cognitive connection between what they observe and what they know (Reason)</li> <li>- Students are unable to use mathematical expressions or symbolic language to explain and justify geometric relationships in connection with visual representations such as diagrams or figures (Representation)</li> </ul>	<ul style="list-style-type: none"> <li>- Students cannot distinguish between what they know (data) and what they do not know (questions), so they do not understand the questions well and tend only to rewrite the given problems (Recognition).</li> <li>- Students are unable to determine whether the information on the given problem is adequate (Recognition).</li> <li>- Students are unable to identify what conditions must be met to solve the problem (Recognition).</li> <li>- Students attempt to comprehend and restate the problem in a more operational (solvable) form (Reason).</li> <li>- Students attempt to devise or consider a problem-solving strategy (Reason).</li> <li>- Student cannot create geometrical shapes to clarify the problems (External Representation)</li> </ul>
Medium	<ul style="list-style-type: none"> <li>- Students demonstrate a good understanding of the problem and are able to devise appropriate problem-solving procedures, although they may make minor errors during execution (Properties).</li> <li>- Students are able to follow and apply problem-solving steps based</li> </ul>	<ul style="list-style-type: none"> <li>- Students thoroughly understand the problem and can devise problem-solving procedures or make minor errors (Recognition).</li> <li>- Students are able to solve problems according to the given problems, but they are incomplete (Recognition).</li> </ul>

on the given information, but their responses are sometimes incomplete or lack final solutions (Properties).

- Students can devise correct procedures but occasionally make symbolic or calculation-related mistakes, such as omitting replacement signs or misapplying formulas (Reason).

- Students understand the mathematical process but may omit parts of symbolic or verbal expressions in their written responses, often due to the belief that these elements are unnecessary (Representation).

- Students understand the problem well and can devise problem-solving procedures, but some replacement signs are missing during the calculation procedure or when substituting the formula (Reason).

- Students omit some parts of symbols and verbal expressions when writing because they think using them is unnecessary (External representation).

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High

- Students are able to grasp underlying mathematical concepts and perform accurate mathematical procedures in connection with visual representations (Properties).

- Students consistently re-examine their solution process and final results, showing reflective thinking and the ability to verify the correctness of their work (Reason).

- Students demonstrate a clear understanding of the problem, are able to draw appropriate geometrical figures, devise logical problem-solving strategies, and accurately solve the problems based on the given conditions (Reason).

- Students understand the problems well, draw geometrical figures, devise problem-solving procedures, and solve problems according to the given problems (Recognition).

- Students re-examine the procedure and the completion results (Reason).

- Students understand creating geometrical drawings to clarify the given problem skills in performing picture representations (External representation).

- Students are able to grasp mathematical concepts and perform a correct mathematical procedure in relation to VR (Recognition).

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- Students possess strong skills in creating geometrical drawings and using visual representations to clarify mathematical problems (Representation).

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According to the data presented in Table 60, it is clear that students' difficulties in solving the problems are owing to a poor understanding of VR and CU of the problem involving geometry, particularly related to vectors and operations on vector content. It also shows that students in the experimental group show higher improvement than the students in the control group.

#### **4.4.4 Students' Perception of Integrating GeoGebra Software**

According to the interview results, students generally have a positive perception of integrating GeoGebra into teaching and learning geometry, as it enhances their visualization of abstract geometry concepts and makes learning more interactive and engaging. It can improve their understanding and retention of difficult topics. However, some students face challenges, such as technical difficulties and an initial learning curve, which can hinder their experience.

A summary of the students' answers to an interview question related to their perceptions of using GeoGebra in teaching and learning geometry is shown below:

##### **T. What do you think about these test items after using GeoGebra software?**

*All the students responded, "... we found GeoGebra helpful for understanding geometry concepts more clearly. We felt more comfortable solving geometry problems because the software visualized key concepts like point locations, coordinates, and vectors. As we had a short time to learn this dynamic software, we wanted to explore it further for further research. Additionally, we appreciated how the tool displayed coordinates clearly, making learning easier. Based on our short experience using this software, we recommend that teachers integrate GeoGebra into mathematics lessons..."* This indicates the students strongly support the integration of GeoGebra software into teaching and learning mathematics, specifically geometry.

Nonetheless, limited access to devices may create inequality in learning opportunities, and there is a concern that over-reliance on the software could weaken

manual problem-solving skills. While GeoGebra is widely appreciated for making geometry more accessible and dynamic, addressing these challenges is crucial for its effective implementation in education.

## CHAPTER 5: FINDING AND DISCUSSION

This chapter discusses the findings to identify whether the results are the same or different from those of previous research. Moreover, this chapter addresses the uniqueness of the research findings.

After doing the field survey, the findings of this study reveal that the Cambodian students face difficulties in learning geometry due to poor visualization skills, specifically in relation to VR, which leads to poor CU in solving geometry problems for the students at upper secondary school education. Prior research consistently highlights the central role of visualization in supporting students' ability to comprehend and reason about geometric relationships. However, many students lack the necessary visual-spatial skills to manipulate geometric figures mentally, which in turn inhibits their ability to develop deeper CU.

Clements (1998) identified visual-spatial reasoning as a fundamental cognitive skill for geometry learning, particularly in secondary education, where students are expected to move from concrete experiences to abstract reasoning. Without sufficient visualization abilities, students struggle to interpret diagrams, imagine geometric transformations, or construct accurate mental images of three-dimensional objects. These difficulties often manifest in poor performance on tasks that require interpretation, justification, or symbolic explanation of geometric phenomena.

Van Hiele's (1986) theory further explains the progression of geometric thinking, emphasizing that students must pass through hierarchical levels of understanding from visual recognition to informal deduction and formal reasoning. When students are unable to visualize or interpret geometric representations meaningfully, they often remain at the lower levels of this model, unable to transition to higher-order reasoning. This lack of progression restricts their ability to engage in abstract geometric thinking, especially when solving geometric problems requiring a conceptual grasp of vector properties or spatial relationships.

Žakej and Klancar (2022) similarly found that although students might be capable of reproducing visual forms or diagrams, they frequently fail to use these visual representations as tools for reasoning. This finding indicates a disconnection

between VR and CU, where students' use of diagrams remains superficial and does not contribute to their understanding of geometric properties or relationships.

Duval (1999, 2006) reinforces this concern by highlighting the importance of coordinating multiple forms of visual, symbolic, and verbal representation in mathematical learning. Students who rely solely on visual forms without translating them into symbolic reasoning or verbal explanation fail to construct integrated mathematical meaning. This finding shows that an incomplete connection between representations results in conceptual gaps that hinder students from solving complex geometry problems.

While technological tools such as GeoGebra and virtual reality (VR) environments offer promising support for visualization, their impact on CU is conditional. Presmeg (2006) and Zhang et al. (2023) cautioned that technology enhances visual access to geometric ideas but does not automatically improve reasoning unless supported by instructional strategies that guide students through reflective and analytical thinking.

Gal and Linchevski (2010) stressed that VR is particularly critical in geometry compared to other branches of mathematics. Their research found that students with weak visualization skills often experience greater geometry difficulties than algebra, particularly in tasks involving spatial reasoning, such as transformations, vector operations, and three-dimensional constructions.

Through the discussion, previous research converges on the finding that poor visualization skills present a significant barrier to conceptual understanding in upper secondary geometry. Although students may engage with geometric diagrams and figures, the lack of ability to mentally manipulate and reason with these representations limits their success in solving problems. To improve students' conceptual understanding, instruction must go beyond the use of visual tools and foster meaningful connections between visual, symbolic, and verbal forms of mathematical representation. Such integration is essential for supporting the progression of geometric thinking and enhancing students' ability to reason abstractly and solve problems effectively.

Another finding of this study demonstrates that integrating GeoGebra significantly enhanced students' VR and CU related to vectors and operations on

vectors. Students exposed to GeoGebra reported increased clarity in their understanding of abstract concepts, supporting the argument that dynamic geometry software effectively bridges the gap between concrete and abstract mathematical thinking.

This result aligns strongly with previous research. Arcavi (2003) and Clements (1992) argued that DGS concretizes abstract mathematical concepts by enabling learners to visually and interactively explore mathematical relationships. GeoGebra's interactive environment provides dynamic visualization, enabling students to directly observe the effects of vector addition, scaling, and transformations, thus facilitating deeper conceptual engagement.

The enhanced conceptual understanding observed in this study also corroborates Zbiek et al.'s (2007) and Presmeg's (2006) findings on the value of technology in mathematics education. Specifically, these researchers highlighted that interactive technologies foster exploratory learning environments where students can experiment, formulate conjectures, and actively verify their reasoning processes. GeoGebra's intuitive manipulation of vectors and immediate feedback capabilities align with this exploratory approach, enabling learners to continuously test and refine their mathematical thinking.

Moreover, the study's outcomes resonate well with Duval's (1999, 2014) cognitive perspective on mathematical learning. Duval asserted that mathematical understanding deepens significantly when learners integrate multiple forms of representation, particularly visual and symbolic. GeoGebra inherently supports this cognitive process by simultaneously displaying algebraic and geometric representations, thus strengthening students' conceptual connections between symbolic vector equations and visual representations of vector operations.

Furthermore, the present findings support Gal and Linchevski's (2010) position that robust VR skills are particularly crucial at the secondary level, where mathematical abstraction increases notably, especially in topics such as vectors. GeoGebra's visual interface assists students in accurately visualizing abstract vector concepts, reducing their cognitive load and enabling more effective processing of complex information.

Student reports from the current study also align with recent empirical findings. Hwang et al. (2020) emphasized that technology-enhanced visualization promotes clearer conceptualization and reduces cognitive difficulties. Similarly, Zhang et al. (2023) observed that digital interactive platforms significantly decrease students' perceived complexity of abstract concepts, thereby improving mathematical comprehension and retention. The positive responses from students using GeoGebra in this study reinforce these findings, indicating improved confidence and reduced anxiety when approaching abstract mathematical tasks.

While dynamic geometry environments like GeoGebra offer powerful opportunities to enhance students' VR skills, several scholars have cautioned that these tools alone do not guarantee CU. Hollebrands (2003), Sinclair and Yurita (2008), and Jones (2000) argued that without structured instructional guidance, students' interactions with VR in such tools may remain surface-level and lack deeper mathematical reasoning.

Hollebrands (2003) highlighted that students often explore geometric relationships through manipulation in dynamic geometry software but fail to connect these visual observations with formal mathematical reasoning. This disconnect can result in misconceptions or incomplete understanding, as students rely on intuition rather than rigorous analysis.

Similarly, Sinclair and Yurita (2008) observed that while students are often engaged by the dynamic and interactive nature of tools like GeoGebra, this engagement does not always translate into conceptual learning. They emphasized that the teacher's role is critical in designing tasks that prompt students to reflect, reason, and generalize from what they observe.

Jones (2000) further supported this view by noting that formal reasoning must complement visual intuition. Students must be guided to articulate why a geometric property holds true, rather than simply observe its dynamic behavior. Without such scaffolding, students may develop procedural familiarity with the tool but lack a solid conceptual framework to explain geometric relationships.

These scholars collectively stress that technology should serve as a means, not an end, in mathematics education. For dynamic tools to be effective, they must be embedded within well-designed instructional sequences that promote structured

inquiry, encourage the articulation of mathematical reasoning, and ensure alignment between VR and formal CU.

Thus, while GeoGebra is a powerful tool for enhancing the teaching and learning of vectors and operations on vectors, its success relies deeply on structured instructional design and active facilitation to foster meaningful, accurate, and deep mathematical understanding.

The results of this study have important implications for teaching practices in geometry. First, we suggest that integrating dynamic geometry software like GeoGebra into the curriculum can greatly enhance students' understanding of complex geometric concepts because its features and functions are easy for students and teachers to explore in-depth to improve their knowledge and skills. Teachers should consider incorporating such tools into their lessons to provide students with a more interactive and engaging learning experience (Hohenwarter et al., 2009; Zulnaidi & Zamri, 2020). Furthermore, given the persistent challenges in VR and CU, it is crucial for educators to focus on these areas more intensively, possibly through additional exercises and instructional methods that explicitly target these skills.

Additionally, the study points out the importance of developing students' visual-spatial skills from an early stage in their education. As visual perception, visual-spatial reasoning, and VR are interlinked and essential for a solid understanding of geometry, these skills should be nurtured systematically across different educational stages (Jones & Tzekaki, 2016; Žakelj & Klancar, 2022).

The findings of this study are similar to those of previous research that have emphasized the importance of visualization in learning geometry (Elia et al., 2018). For instance, previous studies have shown that students who struggle with visualization often find it challenging to grasp geometric concepts fully, leading to lower performance in mathematics overall (Despina A & Silver, 2004). The use of GeoGebra, as demonstrated in this study, aligns with other research that supports the integration of DGS as a means to improve mathematical understanding and problem-solving abilities (Khold, 2022). However, this study contributes further by providing specific insights into the Cambodian educational context, in which such interventions are narrative and imply results.

## CHAPTER 6: CONCLUSION AND RECOMMENDATION

This chapter summarizes the main findings for each research question and makes recommendations to the relevant stakeholders, such as the ministry, curriculum developers, textbook writers, and implementers, and provides suggestions for future research.

### 6.1 Conclusion

The results of this research reveal the following for each research question:

**RQ1:** According to the literature review, Cambodian students are poor in geometry. Students faced difficulties in problem-solving geometry through VR and CU in relation to vectors and vector operations. Through VR, the students are unable: (1) to identify and interpret visual information, including visual, symbolic, and verbal representations (Recognition); (2) to transform and coordinate different types of picture representations in mind (Mental Operation); and (3) to create geometrical drawings to clarify the problem (External Representation); while in relation to CU student are unable: (1) to explain the mathematical concept following the mathematical procedure, including calculation (Properties); (2) to provide the justifications for the mathematical procedure (Reason), and (3) to use the mathematical expression to give the reason in relation to the visual representation (Representation).

**RQ2.** Through the literature review and the comparison of dynamic geometry software, this study found that integrating GeoGebra software could help students improve their VR and CU abilities to overcome difficulties in learning geometry.

**RQ3.** Based on the analysis of the Cambodian mathematics curriculum, specifically geometry, the result reveals a lack of coherence in four points: (1) The relations of the subdomains, contents and learning outcomes are not clearly related or sequenced. They are not well-organized and unnecessarily repetitive. (2) The contents and sub-contents are poorly ordered and are not age-appropriate, because some contents are addressed as a prerequisite, and other contents have a higher level than the student's grade level. (3) The relations of some contents, sub-contents, and their learning outcomes are not clearly specified, as through a lack of logical or conceptual connections within and across years; and (4) Some of the domains, sub-

domains, contents, sub-contents, and learning outcomes are not corresponds to each other as a whole. The curriculum, which lacks coherence, can lead to teachers' difficulty in organizing the teaching materials and students' difficulty in learning because the relation of the contents, sub-contents and learning outcomes is disconnected and makes the students lack conceptual understanding of subjects

**RQ4.** The interventions of GeoGebra software and curriculum coherence focus on vector and operation on vectors content as experimental lesson reveals a significant difference between pre-and post-tests on visual representation and conceptual understanding, which indicates a large effect size based on the difference between both through the Mann–Whitney U-test and Normalized Gain Score. A significant correlation between visual representation and conceptual understanding also resulted in a moderate to strong correlation. Therefore, students' high understanding of visual representation tends to lead to a high understanding of conceptual understanding, which leads to high achievement in solving geometry problems. Thus, integrating GeoGebra software effectively improves students' visual representation and conceptual understanding in overcoming learning difficulties in solving geometry problems, leading to better achievement.

The interview results, focusing on the perception of integrating GeoGebra software into teaching and learning geometry, revealed that students, teachers, and curriculum developers found GeoGebra to be helpful and felt more comfortable solving geometry problems because the software visualized key concepts like point locations, coordinates, and vectors. Even though they have a short time to learn from it, they also want to explore more for further research. Additionally, they appreciated how the tool displayed coordinates clearly, making learning easier. Based on their experience, they recommend that teachers integrate GeoGebra into their lessons

## **6.2 Recommendation**

According to the study results, GeoGebra software is suitable for integrating into teaching and learning geometry in Cambodian education. However, successful implementation requires the consideration of relevant factors.

Teacher training institutions are essential for equipping teachers with adequate training on effectively using GeoGebra for geometry instruction, both in pre-service and in-service settings. The training contents should include well-developed teaching

material and instructional methods to promote the integration of GeoGebra into mathematics education, particularly geometry. This study's sample lessons, lesson plans, and tasks can be used as sample resources. Additionally, GeoGebra should be integrated into lesson plans to align with the Cambodian national curriculum standards for geometry.

Mathematics teachers, who represent the most critical group, must master geometry content and relevant ICT skills, particularly the GeoGebra platform, to familiarize themselves with its functions more effectively.

Schools should provide more computer labs and ICT instructional materials, encouraging students to explore the use of GeoGebra software to enhance their learning, not only at school but also at home or other accessible places, to improve their knowledge and skills in geometry and problem-solving.

Policymakers and MoEYS should allocate funds to create videos on how to use GeoGebra and upload them as online materials for students to explore geometry and related subjects. This will provide students with ICT learning materials and improve their learning experience and skills.

It is important to note that this study is based on targeting upper secondary schools, which may limit the findings in terms of time. Therefore, conducting more replication and longitudinal studies could yield better results and more information about this topic.

### **6.3 Future Issues**

The results of this study provide a detailed analysis of curriculum coherence through the content-relation aspect. To fulfil the entire process of curriculum coherence, the structural aspect, pedagogical aspect, and external coherence also need to be analyzed coherently in future research.

The difference between teaching with curriculum coherence and teaching without curriculum coherence also impacts student learning achievement and cognitive thinking. Future research should clarify the different impacts of these to see how they differ between teaching with a coherent curriculum and an incoherent curriculum.

The results of this study, with a focus on the content relation, show that for improving student learning achievement (the improvement of the control group), the national mathematics curriculum should be systematically revised to ensure a clear, logical progression of content, sub-contents, and learning outcomes across all grade levels. Priority should be given to aligning each grade's content with students' cognitive development, avoiding age-inappropriate or overly advanced materials. Future research should focus more on teacher quality in adapting to the instruction with the curriculum coherence in the content-related aspect.

A strong alignment is needed between what is taught (curriculum), how it is taught (instruction), and how it is measured (assessment). Learning outcomes should clearly reflect the content being delivered, and assessment tasks should be designed to evaluate both visual representation and conceptual understanding in geometry. This alignment ensures consistency and clarity for both teachers and students.

Teachers should receive targeted training on designing and delivering coherent lessons. This includes understanding how to sequence content, link learning outcomes to instructional strategies, and use dynamic geometry software effectively to enhance both visual and conceptual understanding. Continuous support and follow-up coaching are also essential to sustain these practices.

Cambodia's curriculum development efforts should consider the structure and coherence of high-performing systems, such as Singapore, which was found in this study to exhibit strong content alignment and progression. Local adaptation of these best practices can inform more effective curriculum and instructional design.

While this study focused on vector geometry, future research should explore curriculum coherence in other areas of mathematics. This can help determine whether similar coherence issues exist and how they affect students' learning outcomes across the broader mathematics curriculum.

The research highlighted issues in curriculum coherence. Future studies might explore how national curriculum reforms could systematically integrate dynamic geometry tools like GeoGebra and align them with pedagogical goals, assessment strategies, and teacher training programs.

In relation to curriculum coherence and teaching practices, the study recommends revising the Cambodian mathematics curriculum to ensure logical progression and alignment between content, instruction, and assessment. Instructional materials and textbooks should be redesigned to support coherent lesson planning, especially in geometry. Teachers should receive targeted professional development on coherence-based pedagogy and the effective integration of technology such as GeoGebra. Additionally, curriculum reforms should draw on international best practices from high-performing countries like Singapore. Expanding research into coherence across other mathematics topics is also suggested to guide broader curriculum improvement and enhance student learning outcomes nationwide.

The study focused on the immediate outcomes of using GeoGebra-integrated lessons to improve students' conceptual understanding and visual representation in geometry. However, it does not address the long-term retention of knowledge or how sustainable these improvements are over time. Future research could explore follow-up studies to examine the durability of learning gains.

This study was conducted in selected upper secondary schools in Cambodia. Future research could investigate whether similar improvements can be achieved at different educational levels (e.g., lower secondary or primary) or in other provinces, especially rural or under-resourced areas.

While teachers were trained to implement the GeoGebra-integrated lessons, the study does not deeply analyze the challenges teachers face in adapting to such technological integration. Further research could explore how ongoing professional development and support influence the effective use of dynamic software tools in teaching.

Although the study examined learning outcomes, future research could investigate how such interventions affect students' motivation, attitudes, and equity in access (e.g., students with low digital literacy or from marginalized backgrounds).

As the implementation of GeoGebra requires reliable devices and internet access, future issues may involve studying the infrastructural and logistical barriers to broader implementation, particularly in schools with limited resources.

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## APPENDICES

### Appendix 1: Description of the Six Levels of Mathematics Proficiency in PISA 2022

Level	Description
6	Students can work through abstract problems and demonstrate creativity and flexible thinking to develop solutions. For example, they can recognize when a procedure that is not specified in a task can be applied in a non-standard context or when demonstrating a deeper understanding of a mathematical concept is necessary as part of a justification. They can link different information sources and representations, including effectively using simulations or spreadsheets as part of their solution. Students at this level are capable of critical thinking and have a mastery of symbolic and formal mathematical operations and relationships that they use to clearly communicate their reasoning. They can reflect on the appropriateness of their actions with respect to their solution and the original situation.
5	Students can develop and work with models for complex situations, identifying or imposing constraints and specifying assumptions. They can apply systematic, well-planned problem-solving strategies for dealing with more challenging tasks, such as deciding how to develop an experiment, designing an optimal procedure, or working with more complex visualizations that are not given in the task. Students demonstrate an increased ability to solve problems whose solutions often require incorporating mathematical knowledge that is not explicitly stated in the task. Students at this level reflect on their work and consider mathematical results with respect to the real-world context.
4	Students can work effectively with explicit models for complex concrete situations, sometimes involving two variables, as well as demonstrate an ability to work with undefined models that they derive using a more sophisticated computational-thinking approach. Students at this level begin to engage with aspects of critical thinking, such as evaluating the reasonableness of a result by making qualitative judgements when computations are not possible from the given information. They can select

	and integrate different representations of information, including symbolic or graphical, linking them directly to aspects of real-world situations. Students can also construct and communicate explanations and arguments based on their interpretations, reasoning, and methodology at this level.
3	Students can devise solution strategies, including strategies that require sequential decision-making or flexibility in understanding familiar concepts. At this level, students begin using computational-thinking skills to develop their solution strategy. They are able to solve tasks that require performing several different but routine calculations that are not all clearly defined in the problem statement. They can use spatial visualization as part of a solution strategy or determine how to use a simulation to gather data appropriate for the task. Students at this level can interpret and use representations based on different information sources and reason directly from them, including conditional decision-making using two-way tables. They typically show some ability to handle percentages, fractions, and decimal numbers and to work with proportional relationships.
2	Students can recognize situations where they need to design simple strategies to solve problems, including running straightforward simulations involving one variable as part of their solution strategy. They can extract relevant information from one or more sources that use slightly more complex modes of representation, such as two-way tables, charts, or two-dimensional representations of three-dimensional objects. Students at this level demonstrate a basic understanding of functional relationships and can solve problems involving simple ratios. They are capable of making literal interpretations of results.
1a	Students can answer questions involving simple contexts where all the information needed is present, and the questions are clearly defined. Information may be presented in a variety of simple formats, and students may need to work with two sources simultaneously to extract relevant information. They are able to carry out simple, routine procedures according to direct instructions in explicit situations, which may sometimes require multiple iterations of a routine procedure to solve a problem. They can perform actions that are obvious or that require very minimal synthesis of

	information, but in all instances, the actions follow clearly from the given stimuli. Students at this level can employ basic algorithms, formulae, procedures, or conventions to solve problems that most often involve whole numbers.
1b	Students can respond to questions involving easy to understand contexts where all information needed is clearly given in a simple representation (i.e., tabular or graphic) and, as necessary, recognize when some information is extraneous and can be ignored with respect to the specific question being asked. They are able to perform simple calculations with whole numbers, which follow clearly prescribed instructions defined in short, syntactically simple text.
1c	Students can respond to questions involving easy-to-understand contexts where all relevant information is clearly given in a simple, familiar format (for example, a small table or picture) and defined in a very short, syntactically simple text. They are able to follow clear instructions describing a single step or operation.

**Source:** (MoEYS, 2024)

## Appendix 2: A Sample Lesson Plan on Vector Plane and Vector Space

### Lesson plan

### Mathematics

**Date:** Day Month Year

**Subject:** Mathematics

**Part:** 2

**Grade:** 10

**Domain:** Geometry

**Sub-domain:** Vector

**Chapter:** 1: Vector Plane and Vector Space

1.1. Components of a vector

1.2. Length of vector

**Duration:** 2 hours

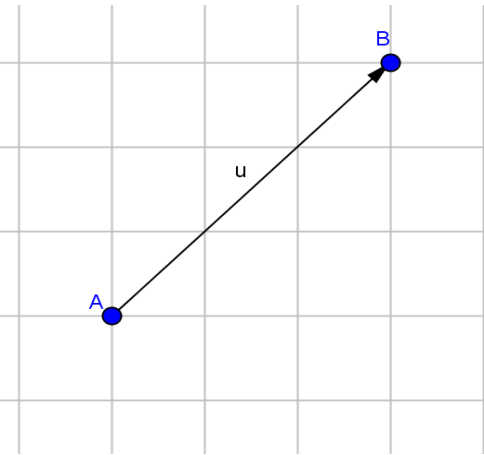
#### I- Objective

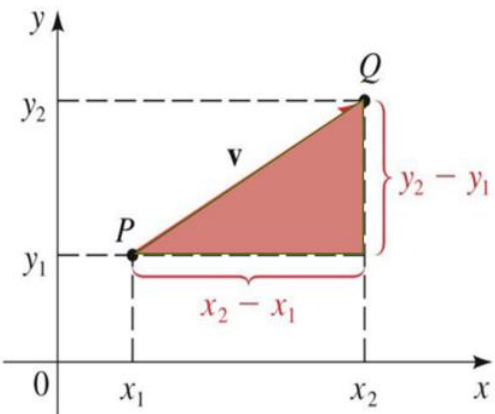
- **Knowledge:** Students can describe a vector plane's notion and relationship followed by teacher instruction.
- **Skill:** Students can correctly compute the length and the relationship of vector planes through teacher instruction and group work.
- **Attitude:** Students solve exercises related to vector planes and daily life problems.

#### II- Materials

- Students: textbook, Learning materials
- Teacher: Textbook, Slides, GeoGebra software

### III- Teaching Procedure

Teacher Activities	Description	Students Activities
<p>Check students' attendants and Classroom management.</p>	<p style="text-align: center;"><b>Step 1:</b> <b>(Classroom Administration)</b></p> <ul style="list-style-type: none"> <li>- Greeting</li> <li>- Attendant</li> <li>- Classroom Management</li> </ul>	<p>The leader (male/Female) reports the attendant to the teacher and classmate.</p>
<ul style="list-style-type: none"> <li>- let a student set two points on the orthonormal system</li> <li>- Another student draws the two connections between both points</li> <li>- Ask the student: What does the picture represent?</li> <li>- Draw the arrow in front of those lines</li> <li>- Ask the students: now, what does the picture represent?</li> </ul>	<p style="text-align: center;"><b>Step 2:</b> <b>(Review the previous lesson)</b></p> <p>Connection with the previous chapter</p> 	<p>Students (Male and Female): - - Sets two points on the board</p> <ul style="list-style-type: none"> <li>- Draw a line connecting from point A to point B.</li> <li>- it is a line</li> <li>- Students observe</li> <li>- individual answer: Vector</li> </ul>

<p>- Demonstrate the figure by using GoGebra Software</p> <p>Talk to students: Today, you can guess what you learned about.</p>		<p>- Observe the figure</p> <p>- Vector</p>
<p>Demonstrate the figure by using GeoGebra Software. Then use the Guided Question:</p> <ol style="list-style-type: none"> <li>1. What is the symbol of the vector plane?</li> <li>2. What is the meaning of a vector <math>\vec{v} = \langle a, b \rangle</math>?</li> <li>3. How is a vector length defined in a plane?</li> </ol> <p>The teacher notes all students' answers on the board.</p> <p>(Misconception:)</p>	<p style="text-align: center;"><b>Step 3.</b> <b>(New Lesson)</b></p> <p style="text-align: center;"><b>Chapter 1. Vector Plane and Vector Space</b></p> <p><b>1. Vector Plane</b></p> <p><b>1.1 Component of vector</b></p> 	<p>- Students provide all possible answers</p> <p>- they are the axis and coordinate of a vector <math> \vec{v}  = \sqrt{a^2 + b^2}</math></p>

<p>- the student may answer that they are the axis and coordinate.</p> <p>-Show a figure of the coordinate system</p> <p>- Ask the student, what can you say about the points <math>P</math> and <math>Q</math> of the vector?</p> <p>- ask individual students: how can you form the vector <math>v</math>?</p> <p>The teacher facilitates the students' answers.</p> <p>- Ask the student, does <math>x_2 - x_1</math> and <math>y_2 - y_1</math> are the axis and coordinate as your answer?</p>		<p><math>P</math> is the initial point and <math>Q</math> is the endpoint.</p> $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$ <p>- No</p> <p>- The length of the orthogonal projection of a vector <math>v</math> on the axis and ordinate</p>
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- facilitate the answer,  $x_1 - x_2$  and  $y_1 - y_2$  called a component of vector  $v$ .

- Demonstrate the figure by using GeoGebra software

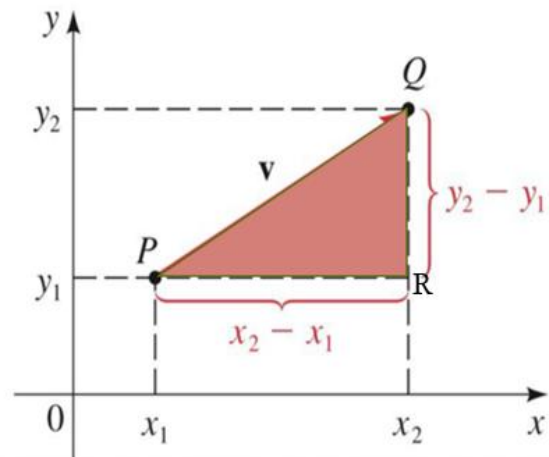
- Set a point  $R$  on the figure, then let the student observe the triangle  $PQR$

- Ask the student individually. What does the triangle represent?

- what does  $PQ$  represent on the triangle  $PQR$ ?

- Divided the students into small groups of 5 to determine the length of the vector  $v$  for 5 minutes

### 1.2 Length of vector



- Students observe and compare the result of their calculation

- Right triangle

- Student answer as an individual: Hypotenus

- Students mixed by gender and set the role and responsibility as a leader, sub-leader, note taker, and presenter

- in the triangle  $PQR$ , has

$$|PQ|^2 = |QR|^2 + |PR|^2$$

$$\Rightarrow |\vec{v}|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow |\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- if vector  $v$  is set in a plan that has an initial point  $P(x_1, y_1)$  and endpoint

$Q(x_2, y_2)$ , then we obtained

$$\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle.$$

<p>- Group's leader report and show the reason for their group's answer (5 minutes/group)</p> <p>- Let the students conclude about the component of the vector and determine the length of a vector.</p>	<p><b>Conclusion</b></p> <p>- if the vector <math>\vec{v}</math> is set in a plan that has an initial point <math>P(x_1, y_1)</math> and endpoint <math>Q(x_2, y_2)</math>, then we obtained</p> $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle.$ <p>The length of the vector <math>\vec{v}</math> determined</p> $ \vec{v}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	<p>The length of the vector <math>\vec{v}</math> determined</p> $ \vec{v}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
<p>- Let the students mixed genders solve the problem on the board.</p>	<p><b>Step 4 (Strengthen knowledge)</b></p> <p>Find <math> \vec{u} ,  \vec{v} ,  2\vec{u} </math> and <math> \frac{1}{2}\vec{v} </math> of</p> <p>1. <math>\vec{u} = 2i + j</math> and <math>\vec{v} = 3i - 2j</math></p>	<p>- Students of mixed gender solve the problem on the board</p> <p>1. <math> \vec{u}  = \sqrt{5},  \vec{v}  = \sqrt{13},</math></p> $ 2\vec{u}  = 2\sqrt{5},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{13}$

<p>-Demonstrate by using GeoGebra</p>	<p>2. <math>\vec{u} = -2i + 3j</math> and <math>\vec{v} = i - 2j</math></p> <p>3. <math>\vec{u} = \langle 10, -1 \rangle</math> and <math>\vec{v} = \langle -2, -2 \rangle</math></p> <p>4. <math>\vec{u} = \langle -6, 6 \rangle</math> and <math>\vec{v} = \langle -2, -1 \rangle</math></p> <p>Demonstrate using GeoGebra Software</p>	<p>2. <math> \vec{u}  = \sqrt{13},  \vec{v}  = \sqrt{5}</math></p> <p><math> 2\vec{u}  = 2\sqrt{13},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{5}</math></p> <p>3. <math> \vec{u}  = \sqrt{101},  \vec{v}  = 2\sqrt{2}</math></p> <p><math> 2\vec{u}  = 2\sqrt{101},  \frac{1}{2}\vec{v}  = \sqrt{2}</math></p> <p>4. <math> \vec{u}  = 6\sqrt{2},  \vec{v}  = \sqrt{5}</math></p> <p><math> 2\vec{u}  = 12\sqrt{2},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{5}</math></p> <p>Students observe and check with their answer.</p>
<p>Please try to solve the exercises that remain in the textbook and let me check your answer for the next time.</p>	<p><b>Step 5. Homework and Advice</b></p>	<p>Students note the lesson</p>

## **Appendix 3: A Sample Lesson Plan on Vector Space**

### **Lesson plan**

**Subject:** Mathematics

**Chapter 1:** Vector Plane and Vector Space

**Sub Chapter:**

1. Vector Plane
2. Vector space
  - 2.1. Components of a vector form
  - 2.2. Length of vector

**Duration 2 hours**

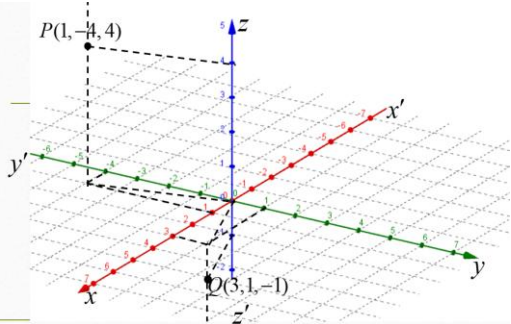
#### **I- Objective**

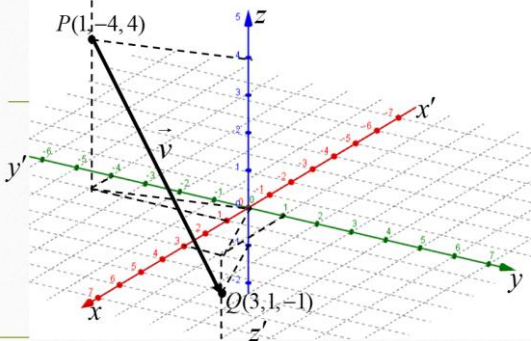
- a. Knowledge: Students can describe a vector space notion and relationship followed by teacher instruction.
- b. Skill: Students can correctly compute the length and the relationship of vector space through teacher instruction and group work.
- c. Attitude: Students solve exercises related to vector space and daily life problems.

#### **II- Materials**

- a. Teacher: Textbook, Slides, GeoGebra software
- b. Students: Learning materials

### III- Teaching procedure

Teacher Activities	Description	Students Activities
Check students' attendance and Classroom management	<b>Step 1: Administrative</b> - Greeting - Attendant - Classroom Management	The leader (male/Female) reports the attendant to the teacher and classmate.
Let students set two points on the orthonormal system.	<b>Step 2: Review the Previous Lesson</b>  Connection with the previous chapter 	- Check previous homework  - Students (boys and girls) set two points on the whiteboard

<p>-Let another student draws a line that connects from <math>P</math> to <math>Q</math></p> <p>- Ask the students: what is the connection from <math>P</math> to <math>Q</math>. Then draw the arrow at the end of the point.</p> <p>- Ask the students: Now, what is the figure look like?</p> <p>What did we learn today?</p>		<p>-Draw a line connecting from <math>P</math> and <math>Q</math></p> <p>- Line</p> <p>- The students observe</p> <p>- Students answer: It is a vector</p> <p>- Vector Space</p>
<p>Demonstrate the figure by using GeoGebra Software. Then use the Guided Question:</p> <p>4. What is the symbol of the vector space?</p> <p>5. What is the meaning of a vector <math>\vec{v} = \langle a, b, c \rangle</math>?</p>	<p><b>Step 3. New Lesson</b></p> <p><b>Chapter 1. Vector Plane and Vector Space</b></p> <p><b>2. Vector Space</b></p> <p><b>2.1 Component of vector form</b></p>	<p>- Students provide all possible answers</p>

6. How is a vector length defined in a space?

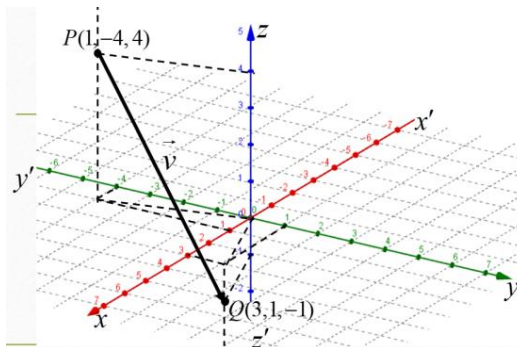
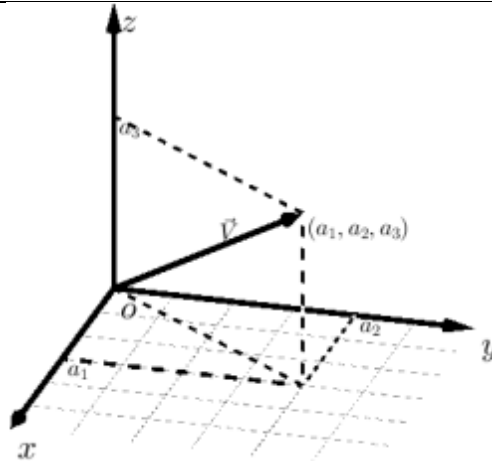
- Noted all possible answers came from students

- Student misconception: Students can think that it is the axis, ordinate, and code vectors

- Show a vector in an orthonormal system

- Ask the student: What can you observe the points  $P$  and  $Q$  on the vector?

- Ask the students: how can set  $\vec{v}$  as a form?



- It is axis, ordinate and code of vector

$$|\vec{v}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

-  $p$  is an initial point and  $Q$  is the endpoint.

$$v = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

- NO

- Teacher facilitates all the answers from students.

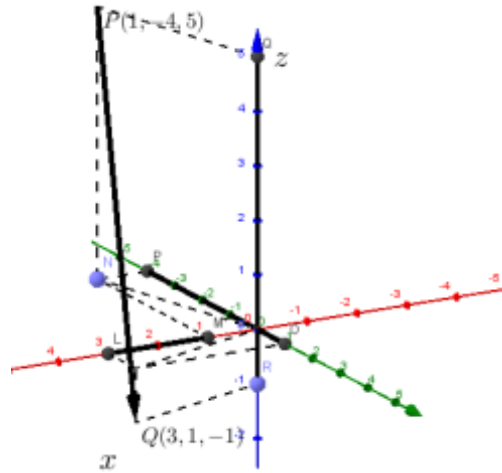
- Ask the students, does

$x_2 - x_1, y_2 - y_1$  and  $z_2 - z_1$  are the axis and ordinate the same as what you have thought above?

- Set a point  $D$  on the picture, then let students observe a triangle  $OAD$

- Ask a student individually: what is the triangle?

- What is  $AD$  in a triangle  $OAD$ ?



## 2.2 Length of vector

It is a length of orthogonal projection of  $\vec{v}$  on axis, ordinate, and code.

- Right triangle

- Student answer individually: Hypotenuse

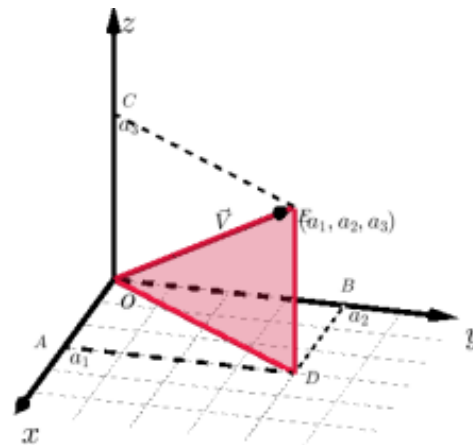
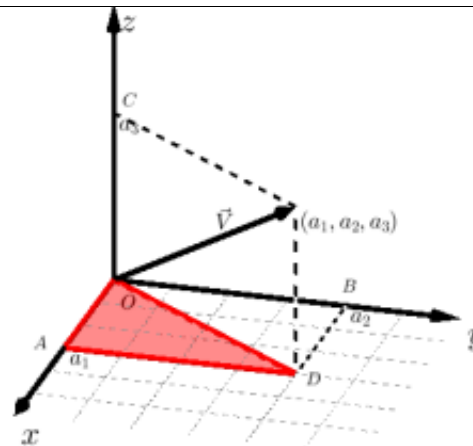
-Set point  $E$  on the picture and let student observe the triangle  $OED$

- Ask the student: what is the triangle represent?

- Divide the students into the small group of five to determine the length of the vector  $\vec{v}$  for five minutes

- Group leader report and describe the reason for their answer (5 minutes for each group)

- Let the student conclude the component of the vector and determine the length of a vector.



- Right triangle

- Student answer individually: Hypotenuse

- Students do as a group by mixed gender and set the role of each group as a leader, sub-leader, note taker, presenter, and members

- In a triangle  $AOD$ , we have

$$|OD|^2 = |OA|^2 + |AD|^2$$

$$\Rightarrow |OD|^2 = a_1^2 + a_2^2$$

	<p><b>Conclusion:</b></p> <p>- If vector <math>\vec{v}</math> set in a plan with the initial point, <math>P(x_1, y_1, z_1)</math> and the endpoint, <math>P(x_2, y_2, z_2)</math> then we</p>	<p>- In a triangle <math>OED</math>, we have</p> $ OE ^2 =  OD ^2 +  ED ^2$ $\Rightarrow  \vec{v} ^2 = a_1^2 + a_2^2 + a_3^2$ $\Rightarrow  \vec{v}  = \sqrt{a_1^2 + a_2^2 + a_3^2}$ <p>- If the vector <math>\vec{v}</math> set in a plan that has the initial point <math>P(x_1, y_1, z_1)</math> and the endpoint, <math>P(x_2, y_2, z_2)</math> then we obtained,</p> $\vec{v} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$ <p>The length of vector <math>\vec{v}</math> determine by</p> $ \vec{v}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
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	<p>obtained <math>v = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle</math>.</p> <p>The length of vector <math>v</math> determine by</p> $ \vec{v}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	
<p>- Let the students of mixed gender solve the problem on the board.</p>	<p><b>Step 4 (Strengthen knowledge)</b></p> <p>1. Find a vector <math>v</math> which has an initial point <math>P</math> and endpoint <math>Q</math> as below:</p> <p>a). <math>P(1, -1, 0)</math> and <math>Q(0, 2, 5)</math></p> <p>b). <math>P(1, 2, -1)</math> and <math>Q(3, -1, 2)</math></p> <p>2. If the vector <math>v</math> has an initial point <math>P</math>. Find the endpoint?</p> <p>a). <math>\vec{v} = \langle 3, 4, -2 \rangle</math> and <math>P(2, 0, 1)</math></p> <p>b). <math>\vec{v} = \langle 0, 0, -1 \rangle</math> and <math>P(0, 1, -1)</math></p> <p>3. Find the length of vector by given</p> <p>a). <math>\vec{u} = \langle -2, 1, 2 \rangle</math></p> <p>b). <math>\vec{v} = \langle 1, -6, 2\sqrt{2} \rangle</math></p>	<p>- Students of mixed gender solve the problem on the board</p> <p>1. <math> \vec{u}  = \sqrt{5},  \vec{v}  = \sqrt{13},</math>  <math> 2\vec{u}  = 2\sqrt{5},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{13}</math></p> <p>2. <math> \vec{u}  = \sqrt{13},  \vec{v}  = \sqrt{5}</math>  <math> 2\vec{u}  = 2\sqrt{13},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{5}</math></p> <p>3. <math> \vec{u}  = \sqrt{101},  \vec{v}  = 2\sqrt{2}</math>  <math> 2\vec{u}  = 2\sqrt{101},  \frac{1}{2}\vec{v}  = \sqrt{2}</math></p> <p>4. <math> \vec{u}  = 6\sqrt{2},  \vec{v}  = \sqrt{5}</math>  <math> 2\vec{u}  = 12\sqrt{2},  \frac{1}{2}\vec{v}  = \frac{1}{2}\sqrt{5}</math></p> <p>Students observe and check their answer.</p>

-Demonstrate by using GeoGebra	Demonstrate using GeoGebra Software	
Please try to solve the exercises that remain in the textbook and let me check your answer for the next time.	<b>Step 5. Homework and Advice</b>	Students take note the lesson

## Appendix 4: Main Survey Lesson Plans

### Lesson Plan Mathematics Grade 10

<b>Date:</b>	Day	Month	Year
<b>Subject:</b>	Mathematics		
<b>Part:</b>	2		
<b>Grade:</b>	10		
<b>Domain:</b>	Geometry		
<b>Sub-domain:</b>	Vector		
<b>Chapter:</b>	1. Vectors and operation on vectors 1.1 Meaning of vector a. Equality vectors b. Opposite vector c. Zero vector a.1 Addition vector		
<b>Duration:</b>	2h:00		

I. **Objective:** After finishing this chapter, the students will be able to

1. **Knowledge:** Describe the meaning of vector, equality of vector, opposite vectors, and Zero vector correctly.
2. **Skills:** Students can identify equality and opposite vectors by practicing examples.
3. **Attitude:** Students cooperate to solve problems with confident

#### II. Materials

- **Student book:** Pages 92-94
- **Teacher guide:** Pages 92-94
- **Long ruler**

III. **Teaching methodology:** Problem-solving method

#### IV. Teaching procedure

<b>Teacher activities</b>	<b>Contents/ Description</b>	<b>Student activities</b>
Checked attendant, Clean, classroom arrangement, and discipline	<p style="text-align: center;"><b>Step 1</b> <b>(Classroom administration)</b></p>	Student representatives report to the class.
<ul style="list-style-type: none"> <li>- Title of the chapter</li> <li>- Post the questions to students to get vector concept</li> </ul>	<p style="text-align: center;"><b>Step 2</b> <b>( Review the previous lesson)</b></p> <ul style="list-style-type: none"> <li>- What is a line, ray, and segment?</li> </ul>	<ul style="list-style-type: none"> <li>- A line is a one-dimensional figure that has length but no width.</li> <li>- Ray is a part of a line that has a fixed starting point but no endpoint.</li> <li>- A segment is a straight line that can be measured.</li> </ul>
<ul style="list-style-type: none"> <li>- A farmer walks from village A to B with the length <math>145\text{ km}</math> , then</li> </ul>	<p style="text-align: center;"><b>Step 3</b> <b>(New lesson)</b></p> <p style="text-align: center;"><b>Vectors and operation on vectors</b></p> <p><b>1. Meaning of vector</b></p>	Find the distance from $A$ to $B$

continues to village C with the length  $189\text{km}$ , which is in the same direction. Calculate the distance from  $A$  to  $B$ .

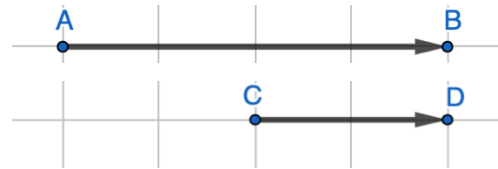
- Definition of vector

- E.g., Construction of vector  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$ , which  $AB \parallel CD$

- **Note.** We can use only one character to name a vector, such as  $\overrightarrow{AB} = \vec{a}$ ,  $\overrightarrow{CD} = \vec{b}$

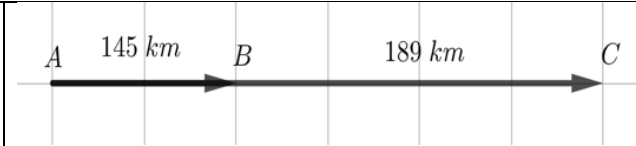
- Vector is the segment which has direction without concerning the position.

- Vector  $\overrightarrow{AB}$  has initial point  $A$  and terminal point  $B$ .



- We obtained that  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  have the same direction.

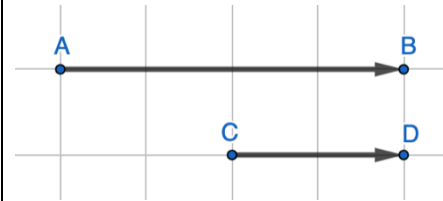
- We obtained  $\overrightarrow{AB} = \vec{a}$ ,  $\overrightarrow{CD} = \vec{b}$



We obtained  $AC = AB + BC$

$$AC = 145 + 189 = 334\text{km}$$

- A vector is a segment whose direction does not concern the position.



- Construct a vector  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$

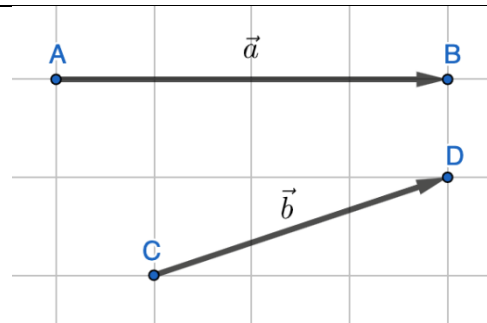
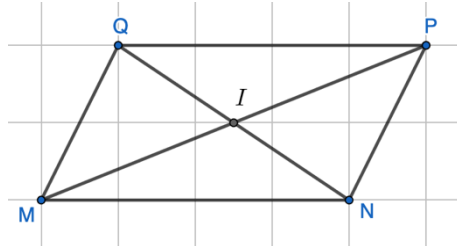
- We obtained a picture as below.

E.g., Given a parallelogram

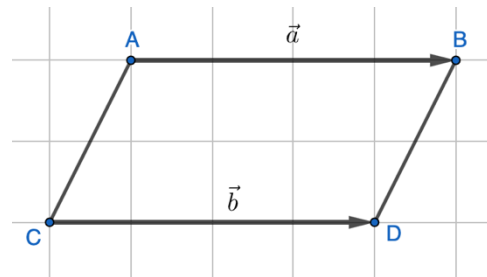
$ABCD$

- A vector  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  have the same direction and equal length

Example: Name equal vectors consisting in the below picture



### A. Equal vector



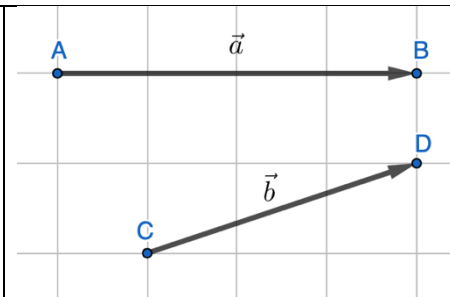
- According to the properties of parallelogram

We obtained  $AB = CD$ ,  $AC = BD$

So,  $\overrightarrow{AB} = \overrightarrow{CD}$ ,  $\vec{a} = \vec{b}$

Definition: two vectors are equal if and only if

. They have the same direction



- We obtained  $\overrightarrow{AB} = \vec{a}$ ,  $\overrightarrow{CD} = \vec{b}$

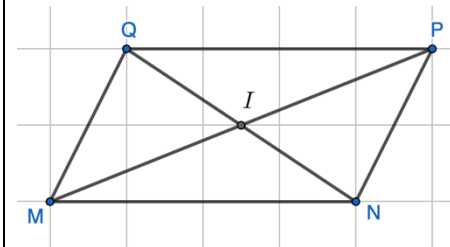
- By properties of parallelogram

$AB = CD$ ,  $AC = BD$

$\overrightarrow{AB} = \overrightarrow{CD}$ ,  $\vec{a} = \vec{b}$

Two vectors are equal if and only if they have the same direction and equal length.

- Equal vectors consist



- . Equal length
- Based on the problem given, an equal vector consists

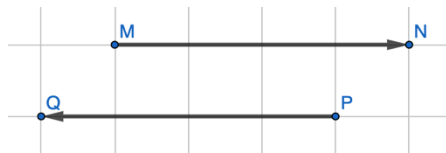
$$\overline{MN} = \overline{QP}, \overline{MQ} = \overline{NP}$$

$$\overline{NI} = \overline{IQ}, \overline{MI} = \overline{IP}$$

### B. Opposite Vectors

- Generally.** Two vectors are equal if and only if they have
- . The same length
  - . Opposite direction

- We have two vectors of equal length



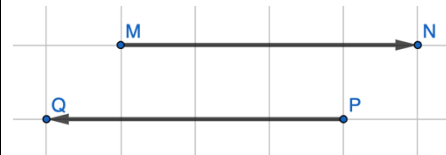
We have  $MN = PQ$

Then, we obtained  $\overline{MN} = -\overline{PQ}$

$$\overline{MN} = \overline{QP}, \overline{MQ} = \overline{NP}$$

$$\overline{NI} = \overline{IQ}, \overline{MI} = \overline{IP}$$

- We have two vectors of equal length



We obtained  $MN = PQ$

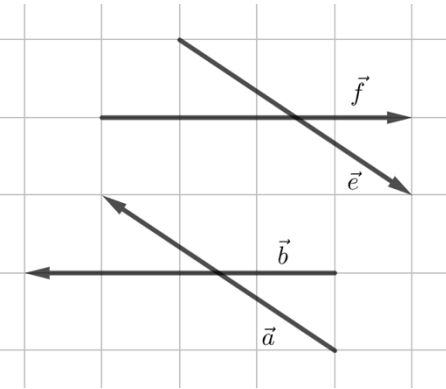
Then  $\overline{MN} = -\overline{PQ}$

**Generally.** Two vectors are equal if and only if they have

- . The same length
- . Opposite direction

**Note:**  $\overline{AB} = -\overline{BA}$

Example: In the below pictures, find opposite vectors.



Example:  $\overrightarrow{AA} = \vec{0}$  ,  $\overrightarrow{BB} = \vec{0}$

We have a rectangular triangle

$ABC$  , which is given

$AB = 4\text{cm}$  ,  $BC = 3\text{cm}$

Calculate  $AC$

**Note:**  $\overrightarrow{AB} = -\overrightarrow{BA}$

- The opposite vectors are:

$\vec{f}$  and  $\vec{b}$  ,  $\vec{a}$  and  $\vec{e}$

a. Zero vector

$\vec{a}$  indicate as zero vector if and only if

$|\vec{a}| = 0$  . Identify zero vector  $\vec{0}$  .

## 1.2. Addition and subtraction vectors

a. Addition vector

**Pythagoras theorem**

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{3^2 + 4^2} = 5\text{cm}$$

Vector can be written as

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

- The opposite vectors are:

$\vec{f}$  and  $\vec{b}$  ,  $\vec{a}$  and  $\vec{e}$

$\vec{a}$  indicate as zero vector if and only if

$|\vec{a}| = 0$  . Identify zero vector  $\vec{0}$  .

- Pythagoras theorem

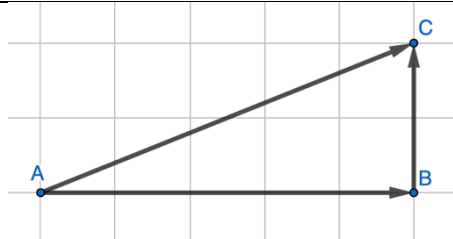
$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{3^2 + 4^2} = 5\text{cm}$$

Vector can be written as

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

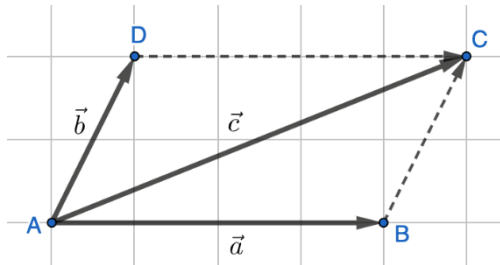
So,  $|\overrightarrow{AB}| + |\overrightarrow{BC}| > |\overrightarrow{AC}|$



So,  $|\overline{AB}| + |\overline{BC}| > |\overline{AC}|$

**Generally**, if we have two vectors  $\overline{AB}$  and  $\overline{BC}$ , the sum of two vectors  $\overline{AB}$  and  $\overline{BC}$  is  $\overline{AC}$ , which is written as  $AC^2 = AB^2 + BC^2$

Parallelogram law of addition

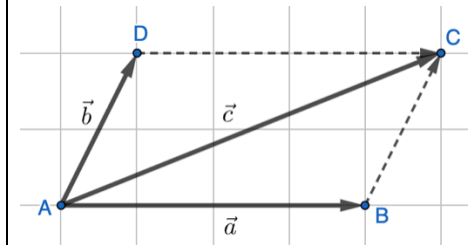


We obtained  $\overline{AB} + \overline{AC} = \overline{AB} + \overline{BD}$   
(Because  $\overline{AC} = \overline{BD}$ )<sup>¶</sup>

We have  $\overline{AB} + \overline{AC} = \overline{AD}$  or  $\vec{a} + \vec{b} = \vec{c}$ .

- Construct the addition of vector  $\vec{F}$  of  $\vec{F}_1$  and  $\vec{F}_2$

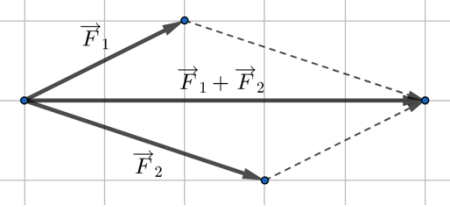
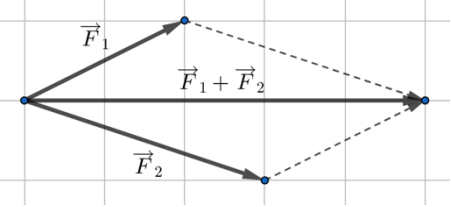
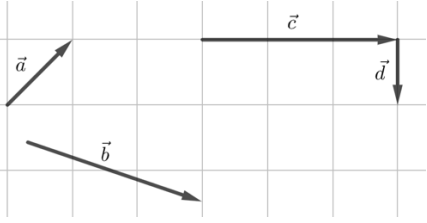
**Generally**, if we have two vectors  $\overline{AB}$  and  $\overline{BC}$ , the sum of two vectors  $\overline{AB}$  and  $\overline{BC}$  is  $\overline{AC}$ , which is written as  $AC^2 = AB^2 + BC^2$



We obtained  $\overline{AB} + \overline{AC} = \overline{AB} + \overline{BD}$   
(Because  $\overline{AC} = \overline{BD}$ )<sup>¶</sup>

We have  $\overline{AB} + \overline{AC} = \overline{AD}$  or  $\vec{a} + \vec{b} = \vec{c}$ .

- Construct the addition of vector  $\vec{F}$  of  $\vec{F}_1$  and  $\vec{F}_2$

<p>- Construct the addition of vector <math>\vec{F}</math> of <math>\vec{F}_1</math> and <math>\vec{F}_2</math></p>		
<p>- Identify the definition of two equal and opposite vectors.</p>	<p style="text-align: center;"><b>Step 4</b> <i>(Strengthening Knowledge)</i></p>	<ul style="list-style-type: none"> <li>- Two vectors are equal if and only if both have the same direction and equal length.</li> <li>- Two vectors are opposite if and only if both have opposite directions and equal length.</li> </ul>
<p>- Identify the sum of <math>\vec{a} + \vec{b}</math>, <math>\vec{c} + \vec{d}</math></p> 	<p style="text-align: center;"><b>Step 5</b> <b>(Homework and Advice)</b></p>	<ul style="list-style-type: none"> <li>- Students taking note of the teacher's advice</li> </ul>

May 01, 2023

**Lesson Plan**  
**Mathematics Grade 10**

**Date:** Day Month Year  
**Subject:** Mathematics  
**Part:** 2  
**Grade:** 10  
**Domain:** Geometry  
**Sub-domain:** Vector  
**Chapter:** 1. Vectors and operations on vectors  
1.2 Addition and subtraction of vectors (Continue)  
a. Addition of Vectors  
b. Subtraction of Vectors

**Duration** 2 Hours

**I. Objective:** At the end of this lesson, students will be able to:

**1. Knowledge:** Students can correctly describe the addition and subtraction of vectors and the multiplication of vectors by a constant.

**2. Skills:** Students can correctly calculate addition and subtraction using the parallelogram, and multiplication with a constant using sample examples.

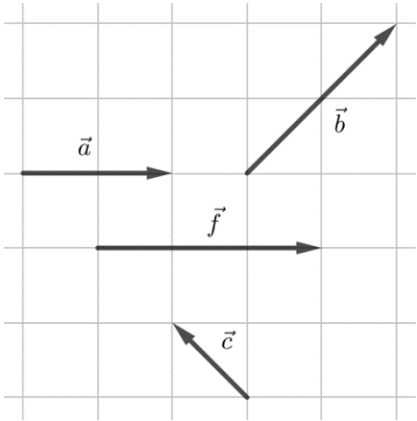
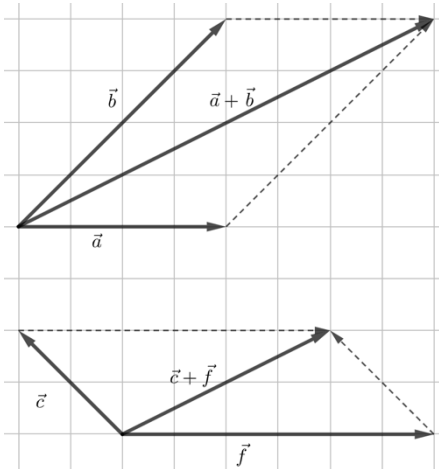
**3. Attitude:** Students cooperated in solving problems with confidence.

**II Materials**

- **Student book:** Pages 95 – 101
- **Teacher book:** Page 95-101
- A long ruler

**III Teaching Methodology:** Using problem-solving methods

#### IV Teaching Procedure

Teacher activities	Contents/ Description	Student activities
Checked attendant, Clean, classroom arrangement, and discipline	<p align="center"><b>Step 1</b> <b>(Classroom Administration)</b></p>	Student representatives report to the class.
<p>– Construct a resultant vector.</p> 	<p align="center"><b>Step 2</b> <b>(Review Lesson)</b></p>	<p>– Construct a resultant vector.</p> 
- Proof of combination and reverse	<p align="center"><b>Step 3</b> <b>New lesson</b> <b>Vectors and operation on</b> <b>vectors (Cont'd)</b></p>	- Taking note of proof of combination and converse

**Example.** Given four different point  $A, B, C, D$

Compare the sum of  $\overrightarrow{AB} + \overrightarrow{BC}$  &  $\overrightarrow{AD} + \overrightarrow{DC}$

In a triangle,  $ABC$  we have

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{AC} + \overrightarrow{BA},$$

then we can write

$$\overrightarrow{BC} = \overrightarrow{AC} + (-\overrightarrow{AB})$$

They said  $\overrightarrow{BC}$  is a subtraction of  $\overrightarrow{AC}$  and  $\overrightarrow{AB}$  ។

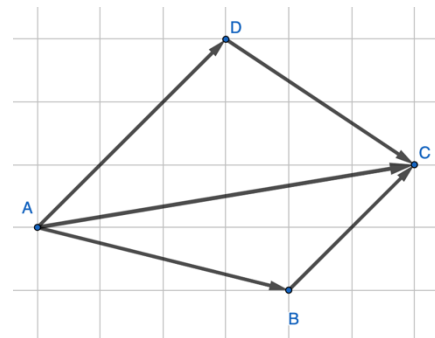
## 1.2 Addition and operation on vectors

- Properties of operation on a vector.

### Properties of vector operation

1.  $\vec{a} + \vec{b} = \vec{b} + \vec{a}$  (Conversion)
2.  $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ , (Combination)
3.  $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$  ,  $\vec{a} + (-\vec{a}) = \vec{0}$

**Note:**  $\vec{c} + (\vec{a} + \vec{b})$  can be written  $\vec{c} + \vec{a} + \vec{b}$  without using brackets.



$$1. \vec{a} + \vec{b} = \vec{b} + \vec{a} \quad (\text{Reverse})$$

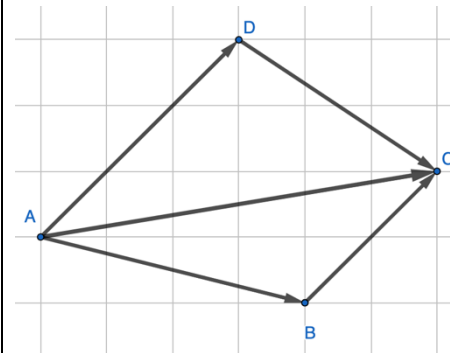
$$2. (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}), \quad (\text{Combination})$$

$$3. \vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a} \quad , \quad \vec{a} + (-\vec{a}) = \vec{0}$$

**Note:**  $\vec{c} + (\vec{a} + \vec{b})$  can be written  $\vec{c} + \vec{a} + \vec{b}$  without using brackets.

▪ Compare

$$\overrightarrow{AB} + \overrightarrow{BC} \quad \& \quad \overrightarrow{AD} + \overrightarrow{DC}$$



$$\text{We obtained } \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \quad (1)$$

$$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC} \quad (2)$$

by (1) and (2)

Example. We have a triangle  $ABC$ .

Find the subtraction of vectors

$$\overline{AB} - \overline{AC} \text{ සහ } \overline{AC} - \overline{BC}.$$

- We have an x-axis of a line number represented by a figure below.



We obtained  $\overline{AC} = 3\overline{AB}$

And  $\overline{PA} = -2\overline{PC}$

We obtained  $\overline{AB} + \overline{BC} = \overline{AC}$  (1)

$$\overline{AD} + \overline{DC} = \overline{AC}$$
 (2)

By (1) and (2)

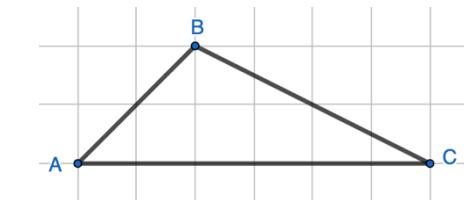
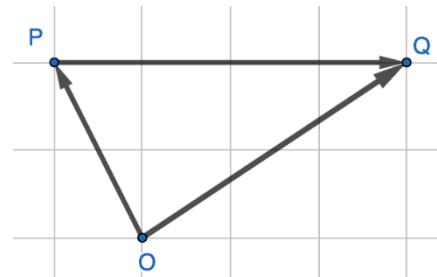
Therefore,  $\overline{AB} + \overline{BC} = \overline{AD} + \overline{DC}$

**b. subtraction of vector**

**In general,**

$$\overline{PQ} = \overline{OQ} - \overline{OP} \text{ for all point}$$

$O$  in the plane.



therefore,  $\overline{AB} + \overline{BC} = \overline{AD} + \overline{DC}$

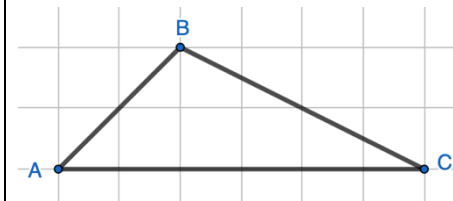
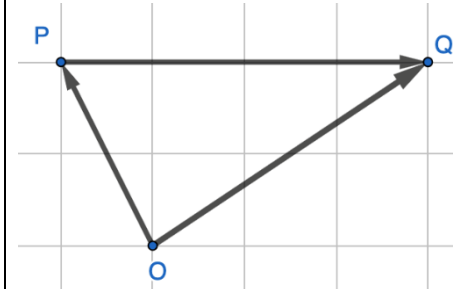
in a triangle,  $ABC$  we have  $\overline{BC} = \overline{BA} + \overline{AC} = \overline{AC} + \overline{BA}$

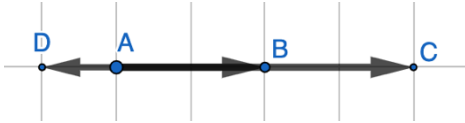
$$\overline{BC} = \overline{AC} + (-\overline{AB})$$

**In general**

$$\overline{PQ} = \overline{OQ} - \overline{OP} \text{ for all point}$$

$O$  in the plane.



<p>Example: Given a vector <math>\overrightarrow{AB}</math>.</p> <p>Construct</p> <p>a. <math>\overrightarrow{AC} = 2\overrightarrow{AB}</math></p> <p>b. <math>\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{AB}</math></p> <p>We have an x-axis of real numbers</p> <p>We obtained <math>\overrightarrow{AC} = 3\overrightarrow{AB}</math></p> <p>and <math>\overrightarrow{PA} = -2\overrightarrow{PC}</math></p>	$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$ $\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AB}$ <p><b>1.3 Multiplication Vector with constants</b></p> <p>a. <i>Multiplication of vector with constants</i></p> <p><b>In general</b>, vector <math>\vec{a}</math> and <math>m</math> identified as <math>m\vec{a}</math>:</p> <ul style="list-style-type: none"> <li>- if <math>\vec{a}</math> is not equal a vector zero <math>\vec{0}</math>, then</li> <li>- if <math>m &gt; 0</math> then <math>m\vec{a}</math> has the same direction as <math>\vec{a}</math> and the length as <math> m \vec{a}</math>.</li> <li>1) If <math>m &lt; 0</math> then <math>m\vec{a}</math> has opposite direction with <math>\vec{a}</math> and length <math> m \vec{a}</math>.</li> <li>2) If <math>m = 0</math>, then <math>m\vec{a}</math> is called <math>\vec{0}</math></li> </ul>	$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$ $\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AB}$ <p><b>In general</b>, vector <math>\vec{a}</math> and <math>m</math> identified as <math>m\vec{a}</math>:</p> <ul style="list-style-type: none"> <li>- if <math>\vec{a}</math> is not equal a vector zero <math>\vec{0}</math>, then</li> <li>- if <math>m &gt; 0</math> then <math>m\vec{a}</math> has the same direction as <math>\vec{a}</math> and the length as <math> m \vec{a}</math>.</li> <li>1) If <math>m &lt; 0</math> then <math>m\vec{a}</math> has opposite direction with <math>\vec{a}</math> and length <math> m \vec{a}</math>.</li> <li>2) If <math>m = 0</math>, then <math>m\vec{a}</math> is called <math>\vec{0}</math></li> <li>- If <math>\vec{a}</math> is equal <math>\vec{0}</math>, then <math>m\vec{a}</math> is called <math>\vec{0}</math>.</li> </ul> 
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**Define the definition of colinear.**

Proof of definition

Example. Calculate

a.  $3\vec{a} + 4\vec{b} - \vec{a}$

b.  $3(\vec{u} + 2\vec{v}) - 3(\vec{u} - 4\vec{v})$

- If  $\vec{a}$  is equal  $\vec{0}$ , then  $m\vec{a}$  is called  $\vec{0}$ .

- Construct

a.  $\vec{AC} = 2\vec{AB}$

b.  $\vec{AD} = -\frac{1}{2}\vec{AB}$



b. *vector colinear*

**Definition.** Given a vector  $\vec{a}$  and  $\vec{b}$  as vector colinear if and only if  $\vec{a}$  and  $\vec{b}$  has the same or opposite direction

We define it as  $\vec{a} // \vec{b}$ .

**. Proof.**

$-(mn)\vec{a} = m(n\vec{a})$ , Combination

$-(m+n) = m\vec{a} + n\vec{a}$ , Separation

**Definition.** Given a vector  $\vec{a}$  and  $\vec{b}$  as vector colinear if and only if  $\vec{a}$  and  $\vec{b}$  has the same or opposite direction

We define it as  $\vec{a} // \vec{b}$ .

**. Proof.**

$-(mn)\vec{a} = m(n\vec{a})$ , Combination

$-(m+n) = m\vec{a} + n\vec{a}$ , Separation



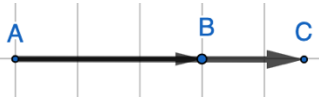
$-m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$ , Separation

- Calculate

a.  $3\vec{a} + 4\vec{b} - \vec{a}$

b.  $3(\vec{u} + 2\vec{v}) - 3(\vec{u} - 4\vec{v})$

$= 3\vec{u} + 6\vec{v} - 3\vec{u} + 12\vec{v} = 18\vec{v}$

	$-m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}, \text{ Separation}$ <p>- Calculate</p> <p>a. <math>3\vec{a} + 4\vec{b} - \vec{a}</math></p> <p>b. <math>3(\vec{u} + 2\vec{v}) - 3(\vec{u} - 4\vec{v})</math></p>	
<p>- Construct a figure below.</p> <p>Given</p>  <p>a. <math>\vec{AC} = -\vec{AB}</math></p> <p>b. <math>\vec{AC} = \frac{3}{2}\vec{AB}</math></p> <p>- Calculate the expression below.</p> <p>a. <math>\frac{3}{2}\vec{a} + 4\vec{b} - 3\vec{b} + \vec{a}</math></p> <p>b. <math>4\left(\frac{1}{2}\vec{u} - 3\vec{v}\right) - \frac{2}{5}(5\vec{u} + \vec{v})</math></p>	<p><b>Step 4</b> <b>(Strengthening)</b></p>	<p>- Construct a vector</p> <p>a. <math>\vec{AC} = -\vec{AB}</math></p>  <p>b. <math>\vec{AC} = \frac{3}{2}\vec{AB}</math></p>  <p>- Calculate</p> <p>a. <math>\frac{3}{2}\vec{a} + 4\vec{b} - 3\vec{b} + \vec{a} = \frac{5}{2}\vec{a} + \vec{b}</math></p> <p>b. <math>4\left(\frac{1}{2}\vec{u} - 3\vec{v}\right) - \frac{2}{5}(5\vec{u} + \vec{v})</math>  <math>= 2\vec{u} - 12\vec{v} - 2\vec{u} - \frac{2}{5}\vec{v} = -\frac{62}{5}\vec{v}</math></p>

<p>. Please do the homework (1) on page 113.</p>	<p><b>Step 5</b> <b>(Homework and Recommendation)</b></p>	<p>- Listen and take notes.</p>
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**Lesson Plan**  
**Mathematics Grade 10**

**Date:** Day Month Year  
**Subject:** Mathematics  
**Part:** 2  
**Grade:** 10  
**Domain:** Geometry  
**Sub-domain:** Vector  
**Chapter:** 1. Vectors and operation on vectors (Cont'd)  
1.3. Multiplication of vectors with constants  
C. Vector unit  
1.4 Coordinate of vectors in a plane  
a. Vector and Coordinate  
b. Coordinate vectors

**Duration:** **2 hours**

- I Objective:** At the end of this part, students will be able to:
1. **Knowledge:** Students can correctly describe a vector unit, coordinate, and length.
  2. **Skill:** Students are able to calculate vector length and coordinate vectors correctly.
  3. **Attitude:** Working as a group, collaborating, and being confident.

**II Materials**

- **Student book:** Page 102-105
- **Teacher Guides:** Page 102-105
- **A Ruler**

**III Teaching Methodology:** Problem-solving Methods

#### IV Teaching process:

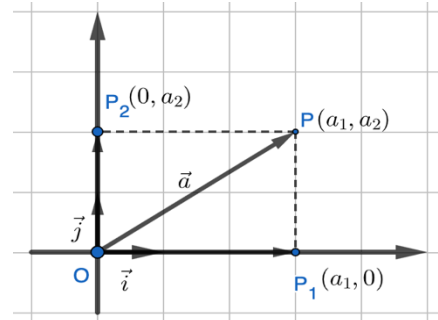
Teacher activities	Contents/ Description	Student activities
Checked attendant, Clean, classroom arrangement, and discipline	<b>Step 1</b> <b>(Classroom Administration)</b>	Student representatives report to the class.
- Calculate the expression below a. $3\vec{u} - \frac{1}{2}\vec{v}$ b. $7(\vec{a} + 3\vec{c} - \vec{b}) + 2(\vec{a} - 7\vec{b})$	<b>Step 2</b> <b>(Review the previous lesson)</b>	- Calculate the expression below a. $3\vec{u} - \frac{1}{2}\vec{u} = \frac{5}{2}\vec{u}$ b. $7(\vec{a} + 3\vec{c} - \vec{b}) + 2(\vec{a} - 7\vec{b})$ $= 7\vec{a} + 21\vec{c} - 7\vec{b} + 2\vec{a} - 14\vec{b}$ $= 9\vec{a} - 21\vec{b} + 21\vec{c}$
- Let students observe a unit vector and proof.	<b>Step 3</b> <b>(New lesson)</b> <b>1.3 Multiplication</b> <i>c. Unit vector</i> <b>Definition</b> <b>Unit vector</b> has the same direction as the given vector but has a magnitude of one unit.	<b>Definition</b> <b>Unit vector</b> has the same direction as the given vector but has a magnitude of one unit.

- Construct a picture in a coordinate plane that has a unit vector. Choose  $P$  as a coordinate which has  $\vec{a} = \overrightarrow{OP}$  ។

-Proof that  $\vec{i} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{|\vec{a}|} \vec{a} \cong \hat{a}$

Unit vector has the same direction as vector  $\vec{a}$  .

**1.4 Coordinate vector in plane**  
**a. vector and coordinate**



Let  $p_1$  and  $p_2$  as orthogonal projection of  $p$  on the x-axis and y-ordinate.

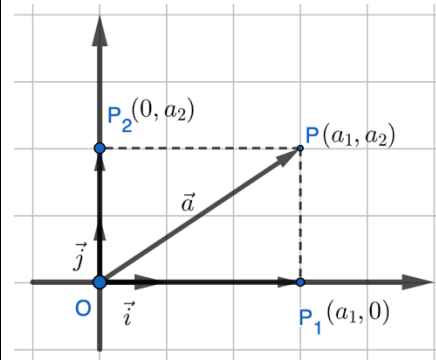
By the law of parallelogram, we obtained  $\vec{a} = \overrightarrow{op} = \overrightarrow{op_1} + \overrightarrow{op_2}$

It can be written  $\vec{a} = a_1\vec{i} + a_2\vec{j}$

-  $a_1$  is the first ordinate of  $\vec{a}$

-Proof that  $\vec{i} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{|\vec{a}|} \vec{a} \cong \hat{a}$

Unit vector has the same direction as vector  $\vec{a}$



Let  $p_1$  and  $p_2$  as orthogonal projection of  $p$  on the x-axis and y-ordinate.

By the law of parallelogram, we obtained  $\vec{a} = \overrightarrow{op} = \overrightarrow{op_1} + \overrightarrow{op_2}$

It can be written  $\vec{a} = a_1\vec{i} + a_2\vec{j}$

-  $a_1$  is the first ordinate of  $\vec{a}$

	<p>– <math>a_2</math> is the second ordinate of <math>\vec{a}</math></p> <p>Therefore, we can write as follows.</p> $\vec{a} = a_1\vec{i} + a_2\vec{j} \text{ or } \vec{a} = (a_1, a_2)$ <p>In general, the length of a vector <math>\vec{a}</math> is</p> $ \vec{a}  = \sqrt{a_1^2 + a_2^2}$ <p>Let <math>p_1</math> and <math>p_2</math> as an orthogonal projection of <math>p</math> on x-axis and y-ordinate</p> <p>By the law of parallelogram</p> <p>We obtained <math>\vec{a} = \vec{op} = \vec{op}_1 + \vec{op}_2</math></p> <p>By <math>op_1(a_1, 0)</math> &amp; <math>op_2(0, a_2)</math></p> <p>By Pythagoras theorem we obtained</p> $ \vec{a} ^2 =  \vec{op}_1 ^2 +  \vec{op}_2 ^2$ $\Rightarrow  \vec{a}  = \sqrt{(0 - a_1)^2 + (a_2 - 0)^2}$ $= \sqrt{a_1^2 + a_2^2}$ <p>Therefore, <math> \vec{a}  = \sqrt{a_1^2 + a_2^2}</math></p>	<p>– <math>a_2</math> is the second ordinate of <math>\vec{a}</math></p> <p>Therefore, we can write as follows.</p> $\vec{a} = a_1\vec{i} + a_2\vec{j} \text{ or } \vec{a} = (a_1, a_2)$ <p>In general, the length of a vector <math>\vec{a}</math> is</p> $ \vec{a}  = \sqrt{a_1^2 + a_2^2}$ <p>Let <math>p_1</math> and <math>p_2</math> as an orthogonal projection of <math>p</math> on x-axis and y-ordinate</p> <p>By the law of parallelogram</p> <p>We obtained <math>\vec{a} = \vec{op} = \vec{op}_1 + \vec{op}_2</math></p> <p>By <math>op_1(a_1, 0)</math> &amp; <math>op_2(0, a_2)</math></p> <p>By Pythagoras theorem, we obtained</p> $ \vec{a} ^2 =  \vec{op}_1 ^2 +  \vec{op}_2 ^2$ $\Rightarrow  \vec{a}  = \sqrt{(0 - a_1)^2 + (a_2 - 0)^2}$ $= \sqrt{a_1^2 + a_2^2}$ <p>Therefore, <math> \vec{a}  = \sqrt{a_1^2 + a_2^2}</math></p>
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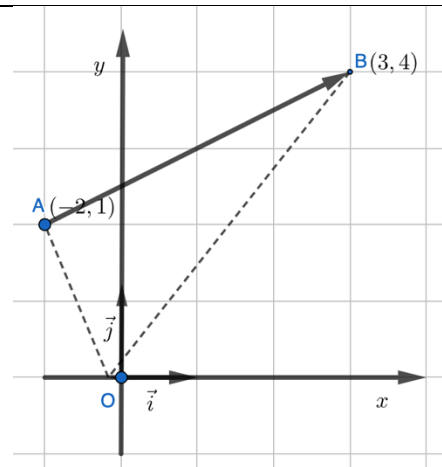
Example. Given points  
 $A(-2,1)$  and  $B(3,4)$  .

a. Calculate the coordinate of

$\vec{AB}$

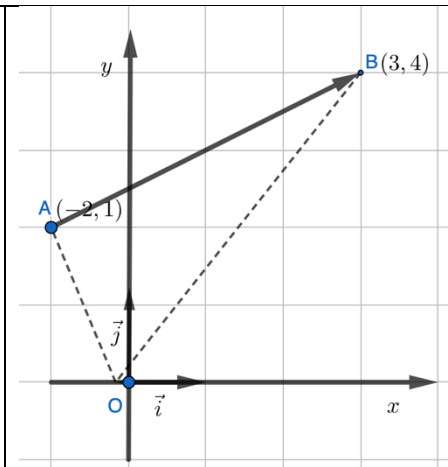
b. Calculate the length of

$|\vec{AB}|$



a.  $\vec{AB} = 5\vec{i} + 3\vec{j} = (5,3)$

b.  $|\vec{AB}| = \sqrt{5^2 + 3^2} = \sqrt{34}$



a. Calculate the coordinate of  $\vec{AB}$

By using a vector addition,

We obtained,  $\vec{AB} = \vec{AO} + \vec{OB}$

$$= (2\vec{i} - \vec{j}) + (3\vec{i} + 4\vec{j})$$

$$= 5\vec{i} + 3\vec{j}$$

Therefore,  $\vec{AB} = 5\vec{i} + 3\vec{j} = (5,3)$

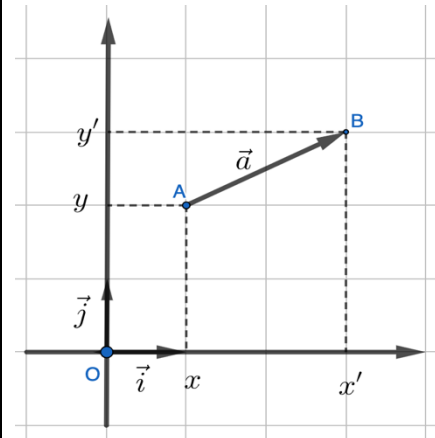
b. Calculate the length of  $|\vec{AB}|$

$$|\vec{AB}| = \sqrt{5^2 + 3^2} = \sqrt{34}$$

Let  $A(x, y)$  &  $B(x', y')$   
 which  $x \in a_1$  &  $y \in a_2$  in the  
 coordinate plane.

Example: Given points  
 $A(2, 3)$  &  $B(-1, 2)$   
 Calculate  $\vec{AB}$ .

**b. Coordinate vector**



Based on the above picture

We obtained,  $\vec{a} = \vec{AB}$

By  $\vec{a} = (a_1, a_2)$ , then

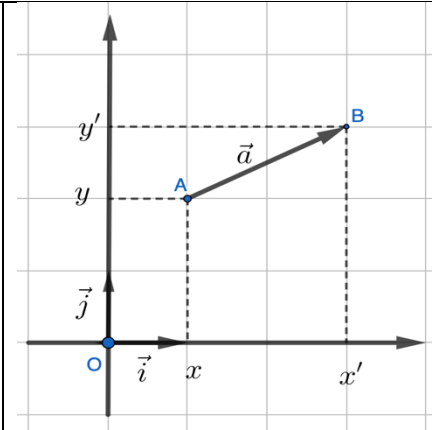
$$a_1 = x' - x \quad , \quad a_2 = y' - y$$

- Calculate  $\vec{AB}$

We obtained

$$\vec{AB} = (-1 - 2, 2 - 3) = (-3, -1)$$

Therefore,  $\vec{AB} = (-3, -1)$



Based on the above picture

We obtained,  $\vec{a} = \vec{AB}$

By  $\vec{a} = (a_1, a_2)$ , then

$$a_1 = x' - x \quad , \quad a_2 = y' - y$$

- Calculate  $\vec{AB}$

We obtained

$$\vec{AB} = (-1 - 2, 2 - 3) = (-3, -1)$$

Therefore,  $\vec{AB} = (-3, -1)$ .

<p>– Given points <math>A(3,4)</math> and <math>B(-4,1)</math>. Calculate <math>\overrightarrow{AB}</math> and <math> \overrightarrow{AB} </math>.</p>	<p style="text-align: center;"><b>Step 4</b> <b>( Strengthening Knowledge)</b></p>	<p>– Calculate <math>\overrightarrow{AB}</math> and <math> \overrightarrow{AB} </math></p> $\overrightarrow{AB} = (-4-3, 1-4)$ $= (-7, -3)$ $ \overrightarrow{AB}  = \sqrt{(-7)^2 + (-3)^2}$ $= \sqrt{58}$
<p>– Given points <math>A(1,1)</math>, <math>B(2,-2)</math> and <math>C(-3,5)</math> <sup>9</sup></p> <p>a. set points <math>A, B, C</math></p> <p>b. Write <math>\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}</math> as a function of <math>\vec{i}</math> and <math>\vec{j}</math></p> <p>c. Calculate the length <math>AB, AC</math></p>	<p style="text-align: center;"><b>Step 5</b> <b>(Homework and advice)</b></p>	<p>.Taking note and listening.</p>

**Lesson Plan**  
**Mathematics Grade 10**

**Date:** Day Month Year  
**Subject:** Mathematics  
**Part:** 2  
**Grade:** 10  
**Domain:** Geometry  
**Sub-domain:** Vector  
**Chapter:** 1. Vectors and operation on vectors (Cont'd)  
c. Calculate the coordinate of the vector  
1.5 Scalar product of vector  
**Duration:** 2 Hours

- I. **Objective:** At the end of this section, students will be able to:
1. **Knowledge:** Students can correctly describe the coordinates of vectors in a plane and the scalar product of vectors.
  2. **Skill:** Students can calculate coordinates and correctly find a scalar product of a vector by using sample examples.
  3. **Attitude:** Students can solve problems as groups with confidence.

**II Materials**

- **Student book:** Page 105 to 108
- **Teacher guide:** Page 105-108
- **Long ruler**

**III Teaching methods:** Problem-solving methods

**IV Teaching procedure:**

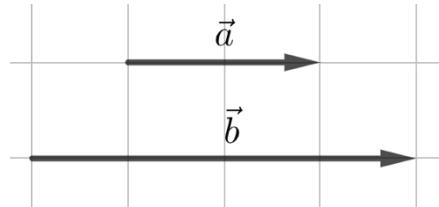
Teacher activities	Contents/ Description	Student activities
Checked attendant, Clean, classroom arrangement, and discipline	<b>Step 1</b> <b>(Classroom Administration)</b>	Student representatives report to the class.
Given points $A(1,2), B(-3,0), C(0,3)$ Calculate $\overline{AB}, \overline{AC}$ as a function of a unit vector $\vec{i}$ and $\vec{j}$ .	<b>Step 2</b> <b>(Review the previous lesson)</b>	- Calculate $\overline{AB}, \overline{AC}$ $\overline{AB} = (-3-1)\vec{i} + (0-2)\vec{j}$ $= -4\vec{i} - 2\vec{j}$ $\overline{AC} = (0-1)\vec{i} + (3-2)\vec{j}$ $= -\vec{i} + \vec{j}$
- Please explain the vector addition of $\vec{a} + \vec{b}$ by using the coordinates of points. If $\vec{a} = a_1\vec{i} + a_2\vec{j}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$	<b>Step 3</b> <b>(New lesson)</b> <b>1.4 Coordinate of vector plane</b> <i>c. The calculation of vectors using coordinate</i> - if $\vec{a} = (a_1, a_2)$ , then $\vec{a} = a_1\vec{i} + a_2\vec{j}$ - if $\vec{b} = (b_1, b_2)$ , then $\vec{b} = b_1\vec{i} + b_2\vec{j}$	- We have known - $\vec{a} = (a_1, a_2)$ , then $\vec{a} = a_1\vec{i} + a_2\vec{j}$ - $\vec{b} = (b_1, b_2)$ , then $\vec{b} = b_1\vec{i} + b_2\vec{j}$ We obtained: $\vec{a} + \vec{b} = (a_1\vec{i} + a_2\vec{j}) + (b_1\vec{i} + b_2\vec{j})$ $= (a_1\vec{i} + b_1\vec{i}) + (a_2\vec{j} + b_2\vec{j})$

<p>Example: Find the Coordinate of vectors <math>\vec{u}, \vec{v}</math> if <math>\vec{a} = (2, -3)</math>, <math>\vec{b} = (-12, 2)</math>, and <math>\vec{c} = (5, 0)</math></p> <p>a. <math>\vec{u} = \vec{a} + 2\vec{b} - 3\vec{c}</math></p> <p>b. <math>\vec{v} = 3(\vec{a} + \vec{b}) - 2(\vec{b} - \vec{c})</math></p>	<p>We obtained</p> $\vec{a} + \vec{b} = (a_1\vec{i} + a_2\vec{j}) + (b_1\vec{i} + b_2\vec{j})$ <p>Therefore,</p> $\vec{a} + \vec{b} = (a_1 + b_1)\vec{i} + (a_2 + b_2)\vec{j}$ <p><b>Calculate vector by using coordinate</b></p> $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ $(a_1, a_2) - (b_1, b_2) = (a_1 - b_1, a_2 - b_2)$ $m(a_1, a_2) = (ma_1, ma_2)$ <p>Example: Find the Coordinate of vectors <math>\vec{u}, \vec{v}</math> if <math>\vec{a} = (2, -3)</math>, <math>\vec{b} = (-12, 2)</math>, and <math>\vec{c} = (5, 0)</math></p> <p>a. <math>\vec{u} = \vec{a} + 2\vec{b} - 3\vec{c}</math></p> <p>b. <math>\vec{v} = 3(\vec{a} + \vec{b}) - 2(\vec{b} - \vec{c})</math></p>	$= (a_1 + b_1)\vec{i} + (a_2 + b_2)\vec{j}$ $\vec{a} + \vec{b} = (a_1 + b_1)\vec{i} + (a_2 + b_2)\vec{j}$ <p>- Calculate <math>\vec{u}, \vec{v}</math></p> <p>a. <math>\vec{u} = \vec{a} + 2\vec{b} - 3\vec{c}</math></p> $= (3, -3) + 2(-12, 2) - 3(5, 0)$ $= (3, -3) + (-24, 4) - (15, 0)$ $= (3 - 24, -3 + 4) - (15, 0)$ $= (-21, 1) - (15, 0)$ $= (-37, 1)$ <p>Therefore, <math>\vec{u} = (-37, 1)</math></p> <p>b. <math>\vec{v} = 3(\vec{a} + \vec{b}) - 2(\vec{b} - \vec{c})</math></p>
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Define the definition of a scalar product

### 1.5 Scalar Product

-  $\theta = 0^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  have the same direction.

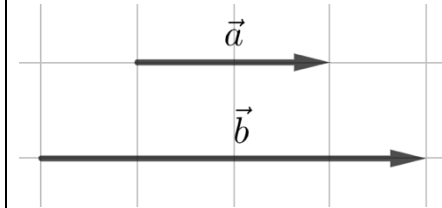


-  $\theta = 180^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  have the same direction.

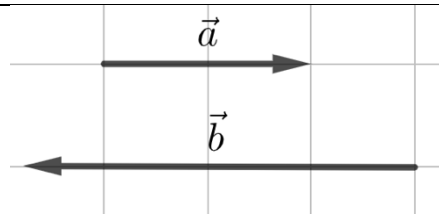
$$\begin{aligned}
 &= 3(2-12, -3+2) - 2(-12-5, 2-0) \\
 &= 3(-10, -1) - 2(-17, 2) \\
 &= (-30, -3) - (-34, 4) \\
 &= (-30+34, -3-4) \\
 &= (-4, -7)
 \end{aligned}$$

Therefore,  $\vec{v} = (-4, -7)$

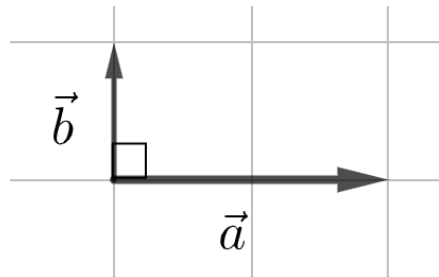
-  $\theta = 0^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  have the same direction.



-  $\theta = 180^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  have the same direction.



-  $\theta = 90^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  are perpendicular.



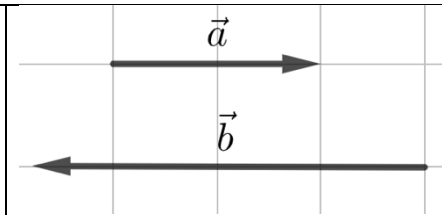
Therefore,  $\theta$  is an angle constructed from the two vectors.

**In general:** Scalar product, if

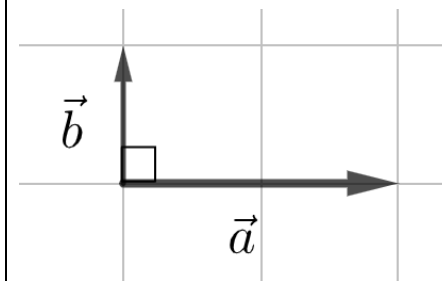
$\theta$  is an angle constructed by the two vectors

$\vec{a}$  and  $\vec{b}$ , the we obtained

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$



-  $\theta = 90^\circ$ , if and only if  $\vec{a}$  and  $\vec{b}$  are perpendicular.



**In general:** Scalar product, if

$\theta$  is an angle constructed by the two vectors

$\vec{a}$  and  $\vec{b}$ , the we obtained

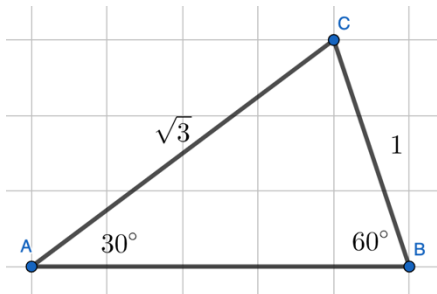
$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Example. Given vectors

$\vec{a}$  and  $\vec{b}$ , which  $|\vec{a}| = 3, |\vec{b}| = 2$

<p>And an angle <math>\theta = 30^\circ</math>, calculate <math>\vec{a} \cdot \vec{b}</math></p> <p>- Construct two perpendicular vectors</p>	<p>- Calculate <math>\vec{a} \cdot \vec{b}</math></p> <p>Using a formula <math>\vec{a} \cdot \vec{b} =  \vec{a}   \vec{b}  \cos \theta</math></p> <p>We obtained <math>\vec{a} \cdot \vec{b} = 2 \times 3 \cos 30^\circ</math></p> $= 2 \times 3 \cos 30^\circ$ $= 2 \times 3 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ <p><b>Note:</b> if <math>\vec{a} = \vec{b}</math>, then <math>\theta = 0^\circ</math></p> <p>Therefore <math>\cos 0^\circ = 1</math>, we obtained</p> $\vec{a} \cdot \vec{a} =  \vec{a}   \vec{a}  \cos 0^\circ =  \vec{a} ^2$ <p>- Scalar product of two perpendicular vectors</p> <p><b>In general.</b> <math>\vec{a} \perp \vec{b}</math>, if and only if <math>\vec{a} \cdot \vec{b} = 0</math></p> <p>- if we have two vectors, <math>\vec{a}</math> and <math>\vec{b}</math></p> <p>Are perpendicular, then <math>\vec{a} \perp \vec{b}</math>, we obtained</p> $\cos \theta = 0.$ <p>Then <math>\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j}</math></p> <p>Therefore, <math>\vec{i} \cdot \vec{j} = 0</math></p>	<p>- Calculate <math>\vec{a} \cdot \vec{b}</math></p> <p>Using a formula <math>\vec{a} \cdot \vec{b} =  \vec{a}   \vec{b}  \cos \theta</math></p> <p>We obtained <math>\vec{a} \cdot \vec{b} = 2 \times 3 \cos 30^\circ</math></p> $= 2 \times 3 \cos 30^\circ$ $= 2 \times 3 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ <p><b>Note:</b> if <math>\vec{a} = \vec{b}</math>, then <math>\theta = 0^\circ</math></p> <p>Therefore <math>\cos 0^\circ = 1</math>, we obtained</p> $\vec{a} \cdot \vec{a} =  \vec{a}   \vec{a}  \cos 0^\circ =  \vec{a} ^2$ <p>- Scalar product of two perpendicular vectors</p> <p><b>In general.</b> <math>\vec{a} \perp \vec{b}</math>, if and only if <math>\vec{a} \cdot \vec{b} = 0</math></p> <p>- if we have two vectors, <math>\vec{a}</math> and <math>\vec{b}</math></p> <p>Are perpendicular, then <math>\vec{a} \perp \vec{b}</math>, we obtained</p> $\cos \theta = 0.$ <p>Then <math>\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j}</math></p> <p>Therefore, <math>\vec{i} \cdot \vec{j} = 0</math></p>
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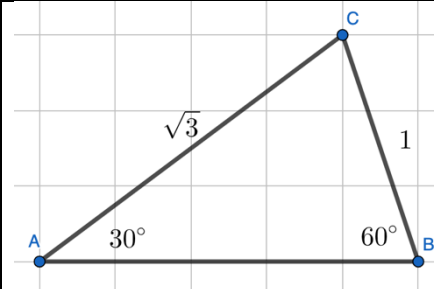
Given a triangle  $ABC$ , which is particularly at  $C$



Calculate the scalar product as below.

- a.  $\overrightarrow{AB} \cdot \overrightarrow{AC}$       b.  $\overrightarrow{CA} \cdot \overrightarrow{CB}$   
 c.  $\overrightarrow{AB} \cdot \overrightarrow{BC}$       d.  $\overrightarrow{AB} \cdot \overrightarrow{CA}$

**Step 4**  
**(Strengthening Knowledge )**



a. Calculate  $\overrightarrow{AB} \cdot \overrightarrow{AC}$ , by applying the formula, we obtained

$$\begin{aligned} \overrightarrow{AB} \cdot \overrightarrow{AC} &= |\overrightarrow{AB}| |\overrightarrow{AC}| \cos 30^\circ \\ &= \sqrt{3} \times 2 \times \frac{\sqrt{3}}{2} = 3 \end{aligned}$$

Calculate  $\overrightarrow{AB} \cdot \overrightarrow{AC}$

Applying a formula

$$\overrightarrow{CA} \cdot \overrightarrow{CB} = |\overrightarrow{CA}| |\overrightarrow{CB}| \cos 90^\circ$$

Therefore,  $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$

Please do the homework in the textbook in sections (Kor) and (Khour).	<b>Step 5</b> <b>(Homework and advice)</b>	- Students taking notes and advice.
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## Appendix 5: Worksheets

School name:.....

Student name.....Sex.....

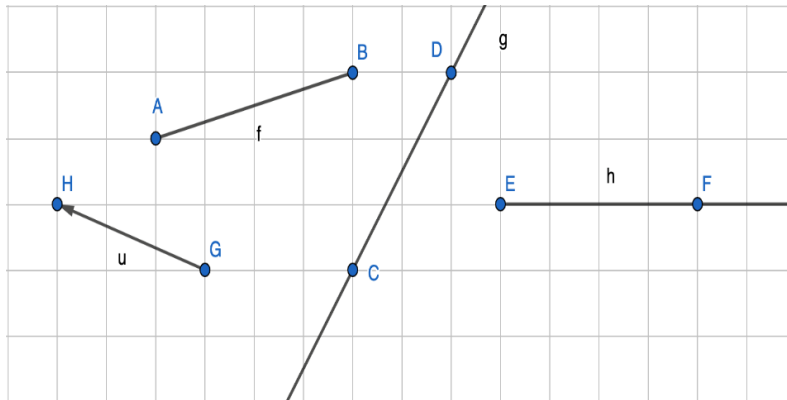
Grade: ..... Age.....

### Worksheet 1

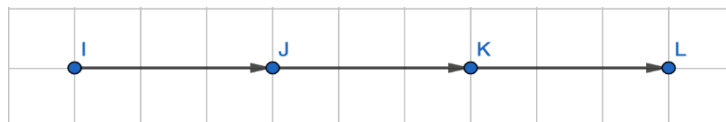
1. A segment with a direction from  $A$  to  $B$  in a plane is called:

- a.  $(AB)$                       b.  $[AB]$                       c.  $\overline{AB}$                       d. Other .....

2. Which one is the representative of the vector in the below picture?



3. In the below picture, how many vectors are represented as equal vectors?



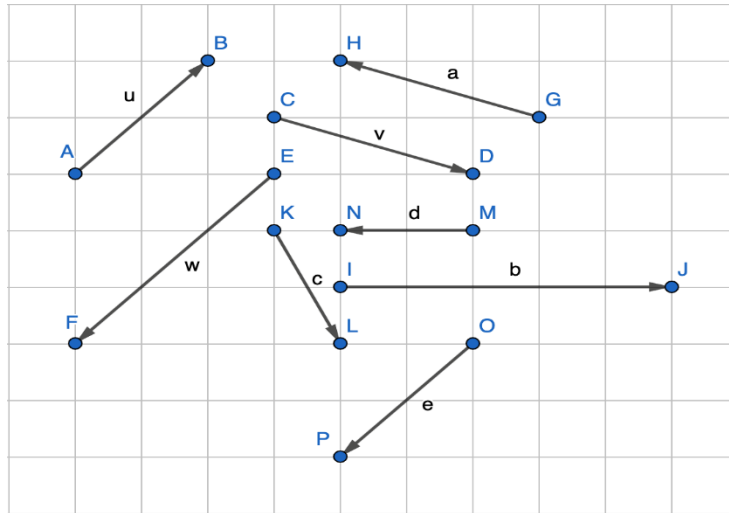
School name:.....

Student name.....Sex.....

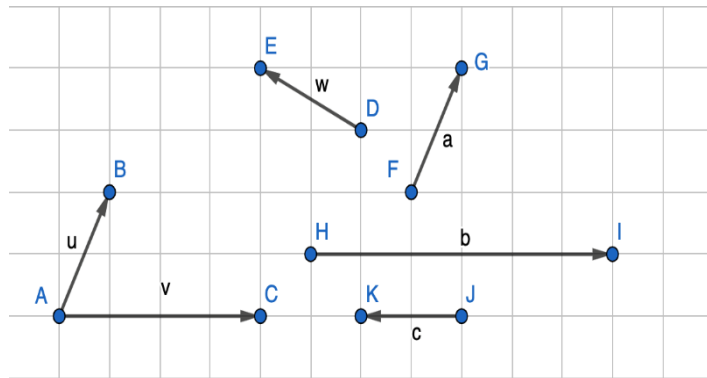
Grade: ..... Age.....

### Worksheet 2

4. Find the opposite vector in the picture below.



5. Determine the addition of vectors using the below picture.



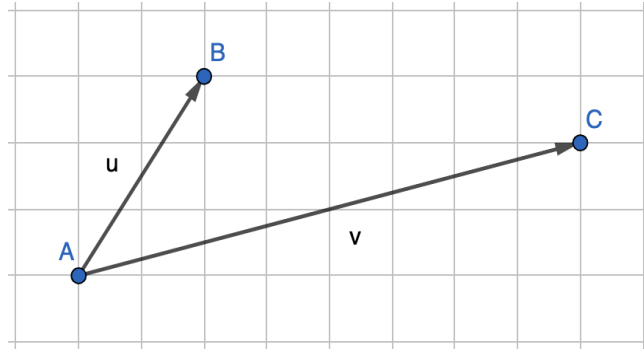
School name:.....

Student name.....Sex.....

Grade: ..... Age.....

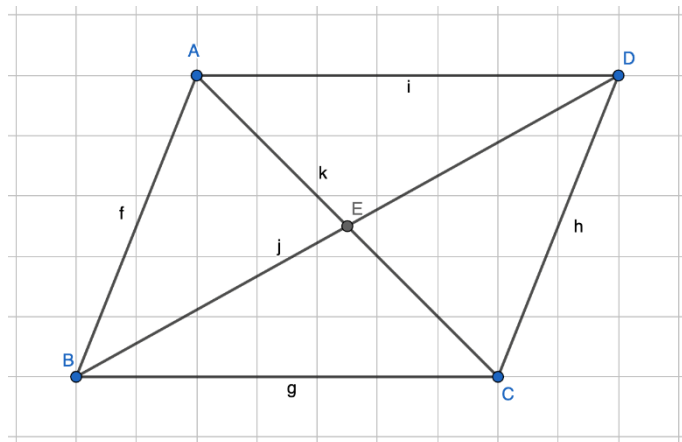
### Worksheet 3

6. Find the subtraction of vectors  $\vec{AB} - \vec{AC}$  and  $\vec{AC} - \vec{AB}$ , as below picture



7. Write the vector addition as a single vector

a.  $\vec{AB} + \vec{BD} + \vec{DC}$  .....



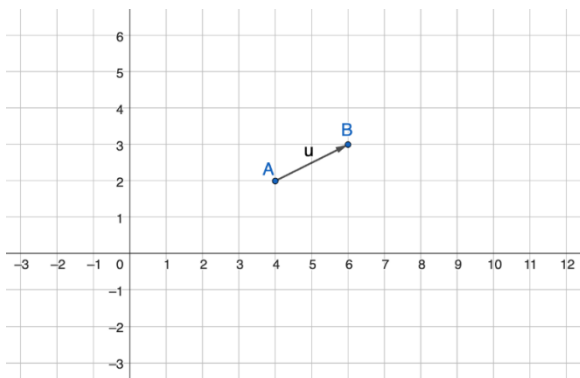
School name:.....

Student name.....Sex.....

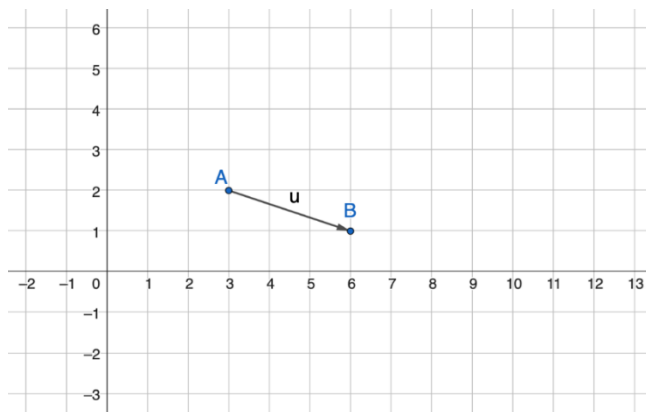
Grade: ..... Age.....

### Worksheet 4

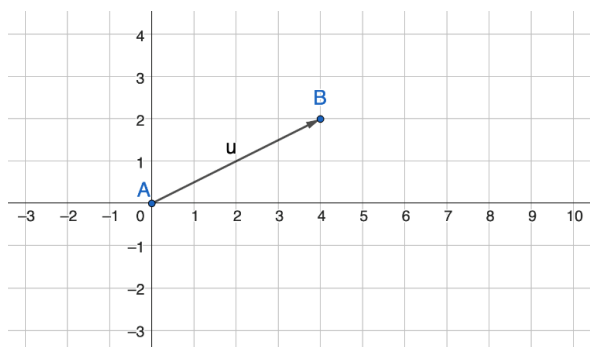
8. Construct a vector  $\overrightarrow{AB} = 2\overrightarrow{AC}$  and  $\overrightarrow{AB} = -3\overrightarrow{AD}$ , as below picture



9. Draw two differences between a vector colinear and a vector  $\overrightarrow{AB}$ , as shown below.



10. Find the coordinate of points  $A, B$  and coordinate of vectors  $\overrightarrow{AB}$ , as below.



School name:.....

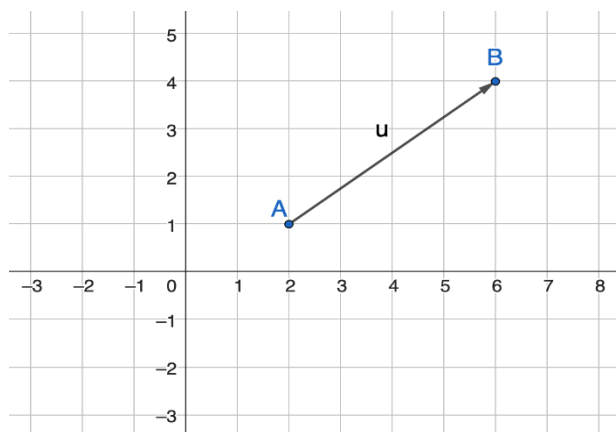
Student name.....Sex.....

Grade: ..... Age.....

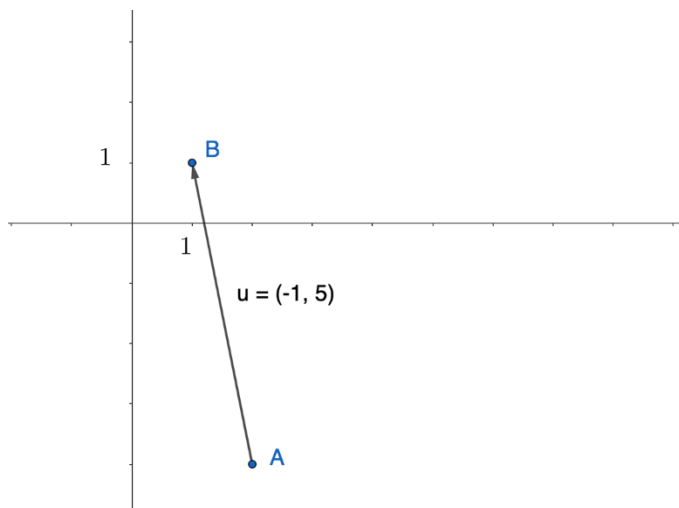
### Worksheet 5

11. Find the coordinate of points  $A, B$  and coordinate of the vector  $\vec{u}$ , as below. Then

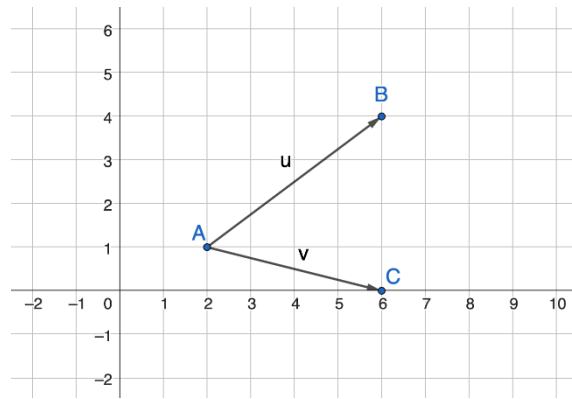
find  $|\vec{u}|$



12. Find the coordinate of a point  $A$  by given  $B(1,1)$  and the coordinate of vector  $\vec{u} = (-1, 5)$ . Then explain on how to get the answer.



13. Draw a vector  $\vec{AD}$  as the addition of vector  $\vec{AB}$  and  $\vec{AC}$ . Then, find the vector coordinate  $\vec{AD}$  and explain how you get the answer.



## Appendix 6: Pilot Test

Name of School:

.....

Name of Student:..... Sex:

.....

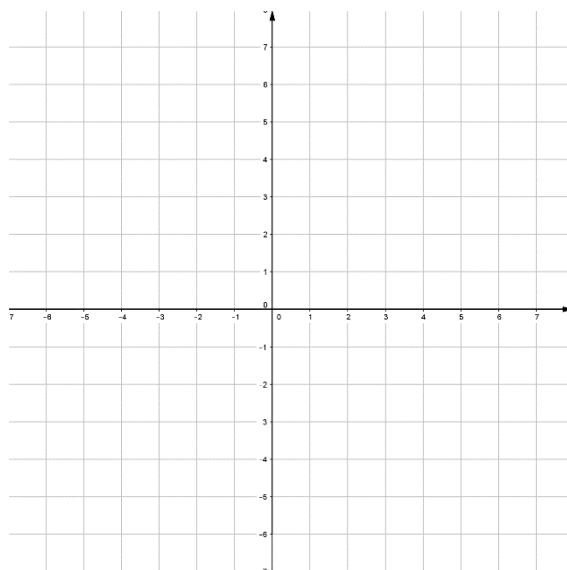
Grade: .....Age:.....

### Instruction:

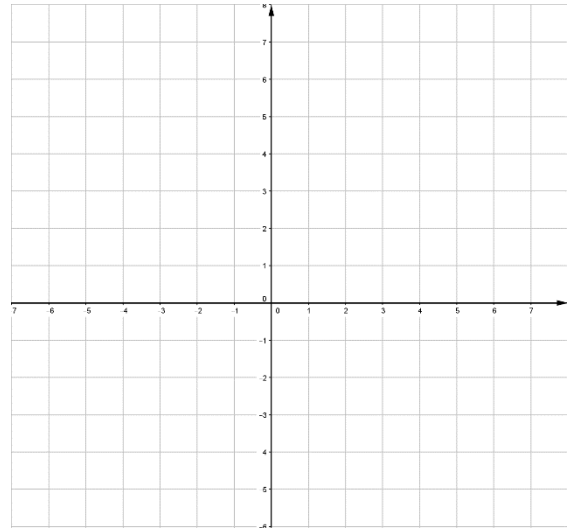
Please write all your answers on this paper only. The answer must be solved by drawing the figure for each solution on your answer sheet. The test takes only 50 minutes.

### Test Items

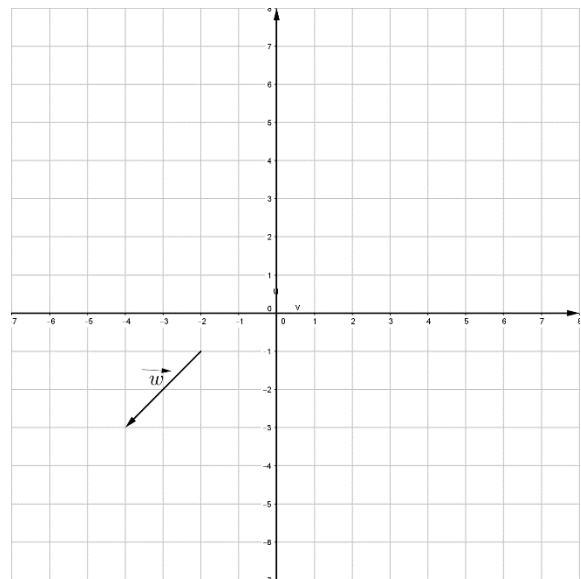
1. Set four points, which are given  $A(3,2)$ ,  $B(-2,1)$ ,  $C(4,-1)$  and  $D(-1,-2)$ . Then, draw the vector  $\overline{AB}$  and  $\overline{DC}$  on the figure below. Please explain how you get the answer.



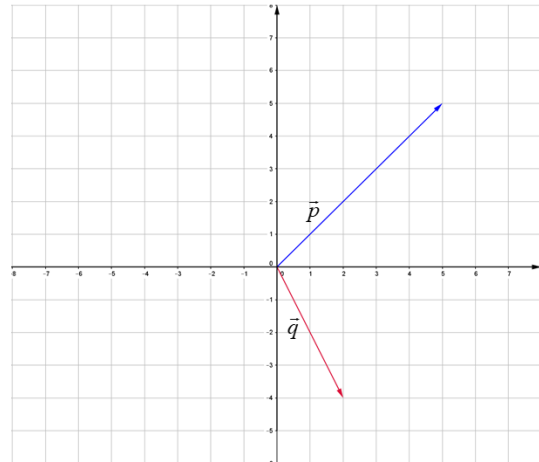
2. Draw the vector with an initial point  $(2,3)$  to the endpoint by giving a vector in component form  $\vec{u} = (2,4)$ , and explain how you get such an answer by showing the graph.



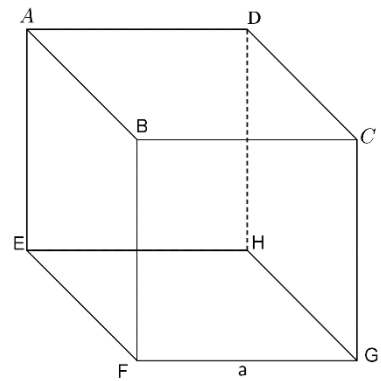
3. Write a vector  $\vec{w}$  in component form to represent the below shape and explain your answer.



4. Graph  $\vec{p} + \vec{q}$  with the same initial point as  $\vec{p}$  and  $\vec{q}$ . Explain how you get the answer.



5. Given a cube that has an edge is  $a$  as shown in the right picture. Find the scalar product of vectors  $\overrightarrow{AF} \cdot \overrightarrow{BG}$ . Please explain by drawing the figure.



## Appendix 7: Pre- and Post-Test

Name of School: .....

Name of Student:..... Sex: .....

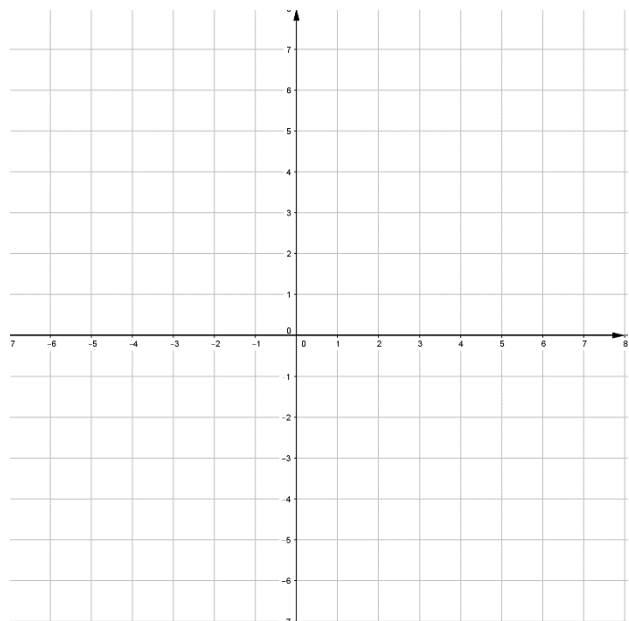
Grade: .....Age:.....

### **Instruction:**

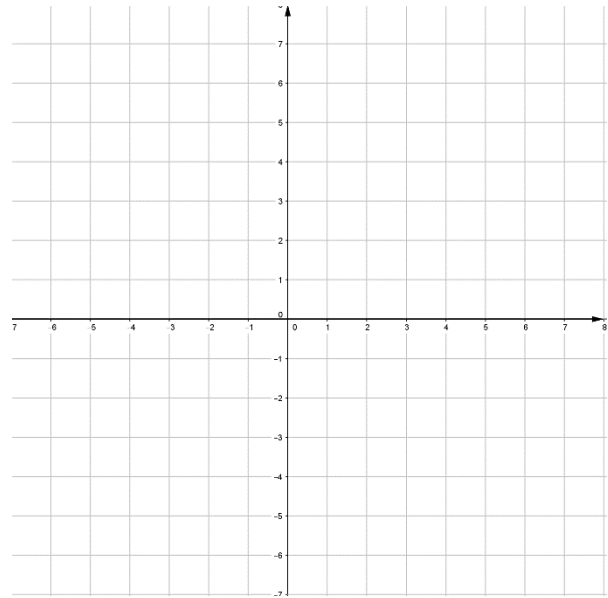
Please write all your answers on this paper only. The answer must be solved by drawing the figure for each solution on your answer sheet. The test takes only 50 minutes.

### **Problem**

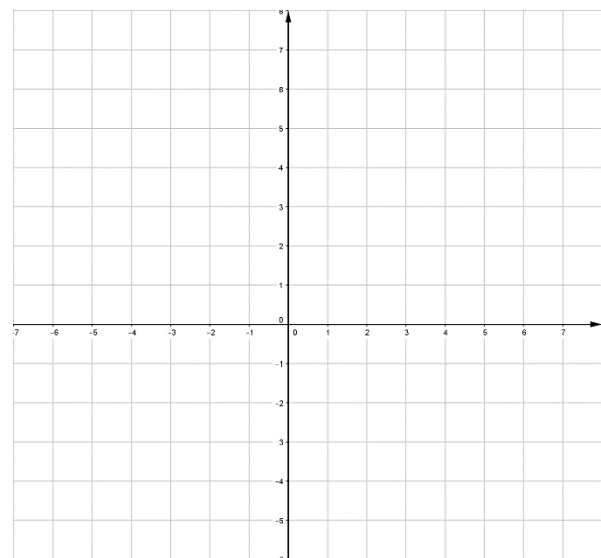
1. Set four points which are given  $A(3,2)$ ,  $B(-2,1)$ ,  $C(4,-1)$  and  $D(-1,-2)$  in the below figure and explain how you get the answer.



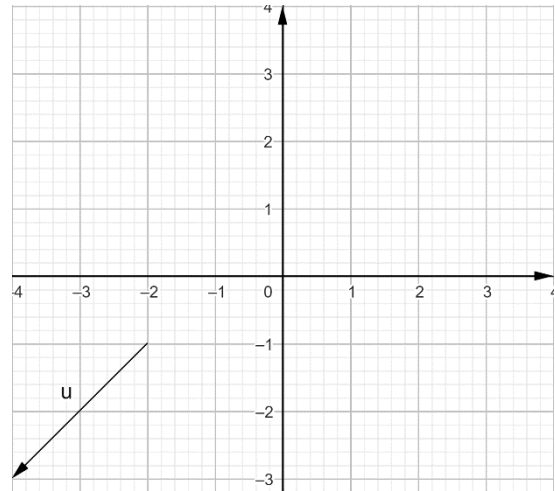
2. Following exercise number 1, draw the vector  $\overline{AB}$  and  $\overline{DC}$ . What can you see on both vectors? Please explain by drawing the figure below.



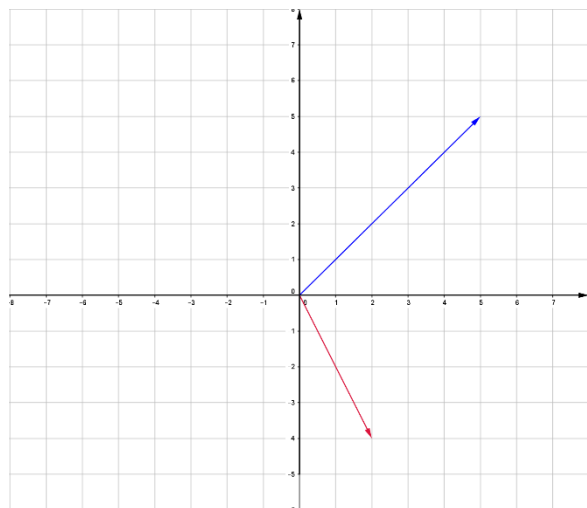
3. Draw the vector with an initial point  $(2,3)$ , then find the endpoint by giving a vector in component form  $\vec{u} = (2,4)$  and explain how you get such an answer by showing the graph.



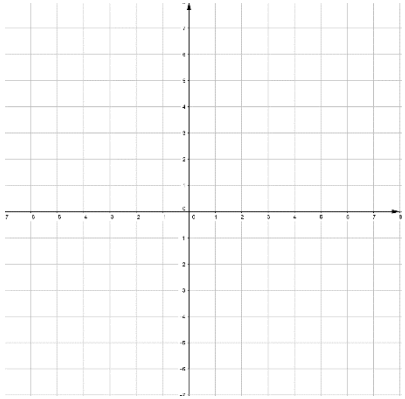
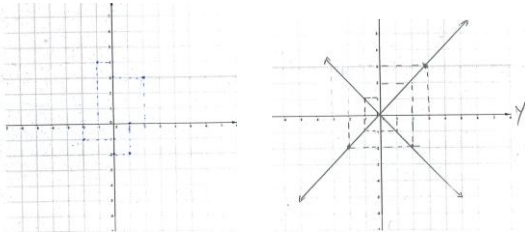
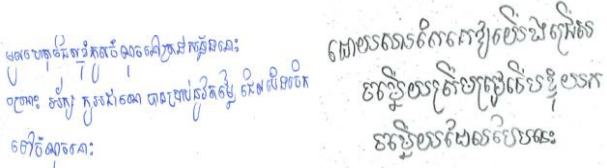
4. Write a vector  $\vec{w}$  in component form to represent the below shape and explain your answer.



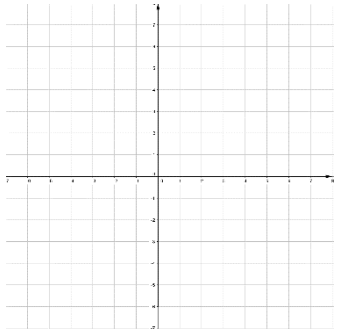
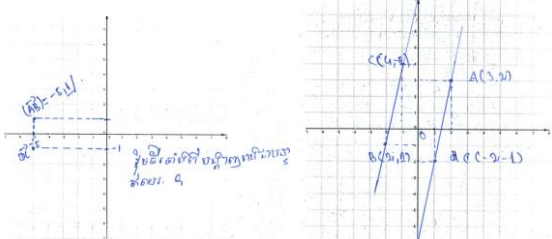
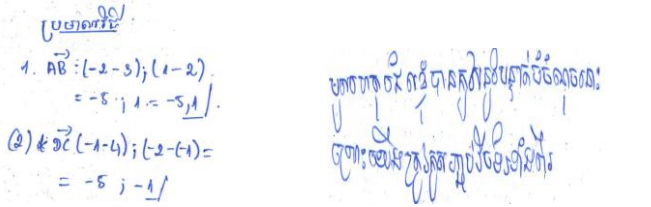
5. Graph  $\vec{p} + \vec{q}$  with the same initial point as  $\vec{p}$  and  $\vec{q}$ . Explain how you got the answer.

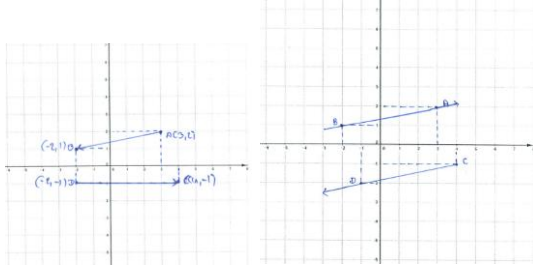


**Appendix 8: Scoring Rubric**

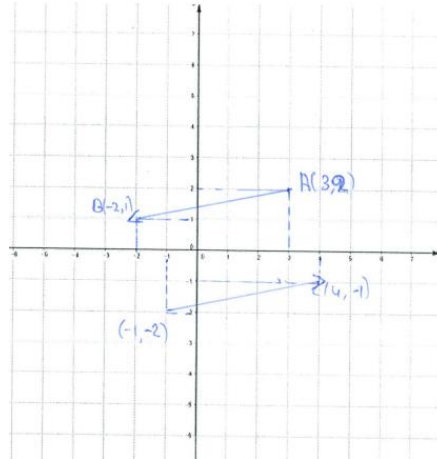
Test Items and Objectives	Response	Visual Representation	Conceptual Understanding	Scoring
<p>1. Set four points which are given A(3,2), B(-2,1), C(4,-1) and D(-1,-2) in the figure below and explain how you get the answer.</p>  <p><b>Objective:</b></p> <p><b>Visual Representation</b> To identify the students' abilities: . to recognize and interpret visual information, including visual, symbolic,</p>	No Answer	There is no attempt to answer the question	There is no attempt to answer the question	0
	Incorrect	<ul style="list-style-type: none"> <li>- The answer provided is completely incorrect in response to the problem given.</li> <li>- Students can plot only one point correctly (Recognize)</li> </ul> 	<ul style="list-style-type: none"> <li>- The answer provided is completely incorrect when performing the mathematical procedure in relation to the visual representation.</li> <li>- Students can explain only one point of their procedure (Properties).</li> </ul> 	1
	Partly Correct	<ul style="list-style-type: none"> <li>- The answer shows some parts of drawing the pictures but is incomplete or contains minor errors (External representation).</li> <li>- Students can plot or construct more than the average of the correct answer (Recognize).</li> </ul>	<ul style="list-style-type: none"> <li>- The answer shows some parts of the correct procedure using mathematical expressions, symbols, and calculations (representation).</li> <li>- Students can explain by addressing the properties of points within three points (Properties).</li> </ul>	2



<p><math>\overline{AB}</math> and <math>\overline{DC}</math>. What can you see on both vectors? Please explain by drawing the figure below.</p>  <p><b>Objective:</b> <b>Visual Representation</b> To identify the students' abilities to create geometrical drawings to clarify the problem and skills in performing picture representations, including visual, symbolic, and verbal presentations.</p> <p><b>Conceptual Understanding</b></p>	<p>Incorrect</p>	<ul style="list-style-type: none"> <li>- The answer provided is entirely incorrect, and the students cannot replot the point or draw the opposite vectors.</li> <li>- The student can replot only one point and can draw the opposite vector correctly in response to the problem given.</li> </ul> 	<ul style="list-style-type: none"> <li>- The answer provided is entirely incorrect, and the students cannot explain how to draw opposite vectors or explain completely incorrect mathematical procedures in response to visual representation.</li> <li>- Students can explain or express the procedure for only one part of the problem or the expressions do not respond to the problem given.</li> </ul> 	<p>1</p>
<p><b>Partly Correct</b></p>	<p>Partly Correct</p>	<ul style="list-style-type: none"> <li>- The answer shows some parts of drawing the opposite vectors but is incomplete or contains minor errors without using symbols and mathematical expressions.</li> <li>- Students can replot the four points and draw the line without mentioning the vector.</li> </ul>	<ul style="list-style-type: none"> <li>- The answer shows some understanding of the concept by describing mathematical procedures and calculation processes, but it is incomplete or contains minor errors.</li> <li>- Students can explain the procedure for the four points, but they cannot explain how to</li> </ul>	<p>2</p>

<p>To describe the students' abilities to:</p> <ul style="list-style-type: none"> <li>. explain the mathematical concept following the mathematical procedure, including calculation (Properties)</li> <li>. give the reason for the mathematical procedure (Reason)</li> <li>. use the mathematical expression to give the reason in relation to the visual representation (Representation).</li> </ul>		<p>- The student drew the opposite vector correctly but missed replotting one point.</p> 	<p>draw the vector using the vector concept or identify the correct procedure for drawing the vector by the definition of vector opposite.</p> <ul style="list-style-type: none"> <li>- Students missed the explanation of some minor points to clarify the drawing procedure.</li> </ul> <p>     + දී ඇති <math>\vec{AB}</math> සිට <math>\vec{DC}</math> ඒකාන්තවෘත්තීයව      + දී ඇති <math>\vec{AB}</math> සිට <math>\vec{DC}</math> ආසන්නවෘත්තීයව   </p> <p>     ඉහත දී ඇති රූපයේ ඇති අංකයන් ඔබ දී ඇති  <math>\vec{AB}</math> සිට <math>\vec{DC}</math> ඒකාන්තවෘත්තීයව      අනුපාතයක් භාවිතයෙන් <math>\vec{DC}</math> වෘත්තීයව      අඳින්න.   </p>	
<p>Draw the vector <math>\vec{AB}</math> and <math>\vec{DC}</math></p>	<p>Correct</p>	<p>- The answer is fully correct and demonstrates a clear understanding of the geometrical drawing to clarify the problem. Replot four points: A, B, C, and D at the proper position and draw the vector with two points <math>\vec{AB}</math> &amp; <math>\vec{DC}</math> following the given problem.</p>	<p>- The answer is entirely correct and describes a clear understanding of the concept by explaining the process using mathematical expressions and the definition of opposite vector concepts following the definition of vectors.</p>	<p>3</p>

- Students are able to use symbols and mathematical expressions



- Students can explain very well on vector position, and opposite vectors by expressing the length of vectors and direction.  
 - Students are also able to explain the procedure of addressing the symbolic and mathematical expression.

+  $\vec{AB}$  និង  $\vec{DC}$  ដាច់ចំនួនដើមគ្នា  
 - ឆ្ងាយ  $\vec{AB}$  និង  $\vec{DC}$  មានទិស  
 ផ្សេងគ្នា តែមានប្រវែងដូចគ្នា  
 ដូចគ្នា :  $\vec{AB} = -\vec{DC}$

The figure shows that

- Vector  $\vec{AB}$  and  $\vec{DC}$  are different direction
- The length of both vectors are equal
- Then vector  $\vec{AB}$  &  $\vec{DC}$  is an opposite vector
- we can write  $\vec{AB} = -\vec{DC}$  or  $\vec{DC} = -\vec{AB}$

**Figure:**

- Set points  $A, B, C, D$  at the right position
- Connect the vector between 2 points  $\vec{AB}$  &  $\vec{DC}$



3. Draw the vector with an initial point  $(2,3)$ , then find the terminal point by giving a vector in component form  $\vec{u} = (2,4)$  and explain how you get such an answer by showing the graph.

No Answer

There is no attempt to answer the question

There is no attempt to answer the question

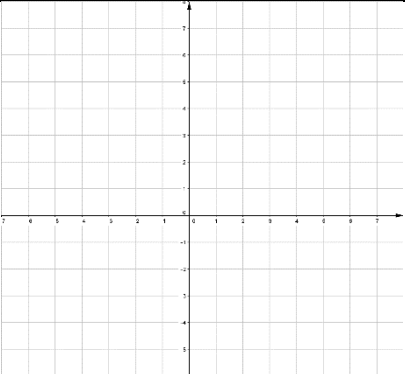
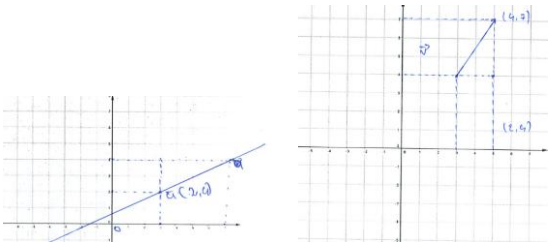
0

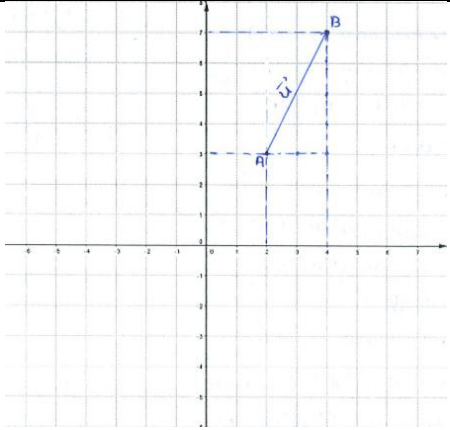
Incorrect

- The answer provided is entirely incorrect. The student cannot draw a vector from the initial point to the terminal point.  
- Students can draw vectors and plot the points but their answers do not respond

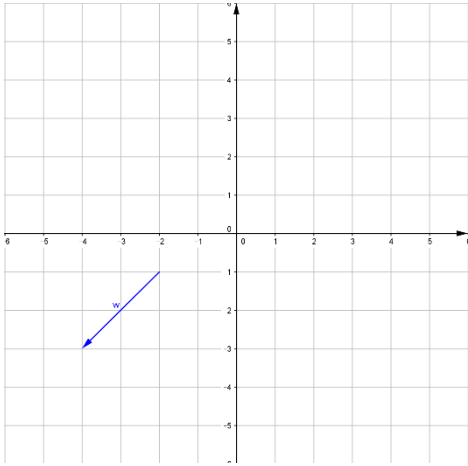
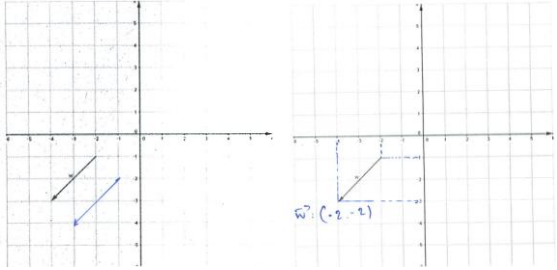
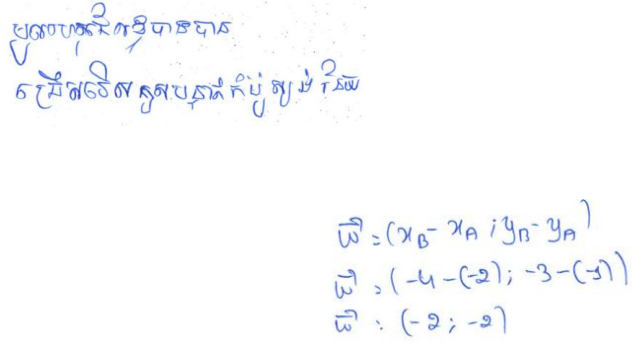
- The answer provided is entirely incorrect. The students cannot explain the concept of finding the terminal point of the vector by using mathematical expressions.  
- Student can explain the result followed by their drawing but the explanations do not respond to the given problem.

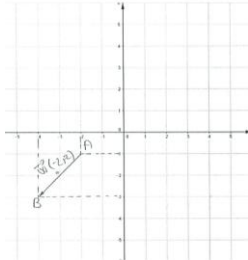
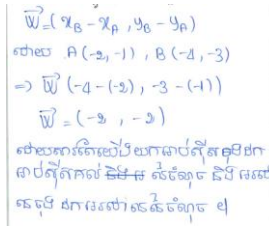
1

		<p>to the problem given or correct very small parts.</p> 	<p>ಸರಿಯಾದ ಕೆಲಸಗಳನ್ನು ಸಹಿಸಿ ಭಾಗಶಃ ಸರಿಯಾದ ಕೆಲಸ</p> <p><math>\vec{u}: B-A = u+A = (2, 3)(2, 4)</math>  <math>A(2, 3) B(4, 7)</math></p>	
<p><b>Objective:</b>  <b>Visual Representation</b>          To identify the students' abilities to create geometrical drawings to clarify the problem and skills in performing picture representations, including visual, symbolic, and verbal presentations.</p>	<p>Partly Correct</p>	<ul style="list-style-type: none"> <li>- The answer shows a drawing of the vector concept that is incomplete or contains minor errors.</li> <li>- Students plot points and use mathematical expressions but either of them is incorrect.</li> <li>- The students could draw the vector from the initial point to the terminal point but did not use symbols and mathematical expressions to clarify the picture.</li> </ul>	<ul style="list-style-type: none"> <li>- The answer shows some concept understanding but is incomplete or contains minor errors.</li> <li>- The students can clearly explain the answer but do not use symbols and mathematical procedures.</li> <li>- The procedure of the calculation is not completely correct or contains some missed calculations.</li> </ul>	<p>2</p>

<p><b>Conceptual Understanding</b></p> <p>To describe the students' abilities to grasp mathematical concepts and perform correct mathematical procedures in relation to visual representation.</p>			<p style="text-align: center;"><u>ઉત્તર</u></p> <p>બેજન્ટ ઇનેલ શ્રેણી સંબંધિત ઉત્તરો.</p> <p>૦ વેક્ટરના નોંધણી સમયે વેક્ટરના પૂર્ણ પદો છે.</p> <p>૨ સંકેત યુક્તિની વેક્ટરના નોંધણી સમયે.</p> <p>પદો ૪ છે. ૭ યુક્તિ વેક્ટરના નોંધણી સમયે.</p> <p>વેક્ટરના નોંધણી સમયે <math>(4, 7)</math>.</p>	
	<p>Correct</p>	<ul style="list-style-type: none"> <li>- The answer is fully correct and demonstrates a clear understanding of the vector concepts. Based on the problem given.</li> <li>- Students can draw a vector starting from an initial point (2, 3) to a terminal point (4, 7).</li> <li>- Students can express mathematical expressions on the figure</li> </ul>	<ul style="list-style-type: none"> <li>- The answer is fully correct and demonstrates a clear understanding of the concept.</li> <li>- The students can explain how to find the terminal point by giving vector component form.</li> <li>- Students can explain by writing the procedure using symbols and mathematics procedures to clarify the process to get the final answer in relation to visual representation.</li> </ul>	<p>3</p>

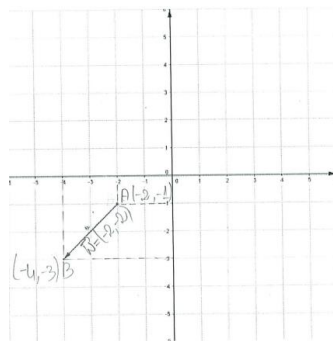


<p>4. Write a vector <math>\vec{w}</math> in component form to represent the below shape and explain your answer.</p>  <p><b>Objective:</b> <b>Visual Representation</b> To identify the students' abilities to create geometrical drawings to clarify the problem and skills in performing picture presentations, including visual, symbolic, and verbal presentations.</p>	No Answer	There is no attempt to answer the question	There is no attempt to answer the question	0
	Incorrect	<ul style="list-style-type: none"> <li>- The answer provided is completely incorrect and does not follow the given problem.</li> <li>- Students cannot draw vectors <math>\vec{w}</math> in component form for the problem given.</li> <li>- Students can not verify the vector component.</li> </ul> 	<ul style="list-style-type: none"> <li>- The answer provided is completely incorrect.</li> <li>- Students cannot express mathematical concepts of how to find a vector <math>\vec{w}</math> in component form using mathematical operations or procedures in relation to visual representation.</li> <li>- Student's answers do not respond to the problem given.</li> </ul> 	1
	Partly Correct	- The answer shows some concepts for drawing a vector in component form but	- The answer shows some concept understanding but is incomplete or contains minor errors.	2

<p><b>Conceptual Understanding</b></p> <p>To describe the students' abilities to grasp mathematical concepts and perform correct mathematical procedures in relation to visual representation.</p> <p>Write vector component</p>		<p>is incomplete or cannot clarify how to get the vector <math>\vec{w}</math> in component form.</p> <ul style="list-style-type: none"> <li>- Student notified the coordinate system of vector components by using the initial point and endpoint.</li> <li>-Students missed some parts of mathematical expressions on the figure in the right ways.</li> </ul> 	<ul style="list-style-type: none"> <li>- Students can explain by setting the initial point and terminal point but cannot write the procedure for getting a vector <math>\vec{w}</math> in component form.</li> <li>- Students planned the calculation procedure on how to get the result of the vector component form but still missed some points such as mathematics expressions or symbols.</li> </ul> 	
	<p>Correct</p>	<ul style="list-style-type: none"> <li>- The answer is fully correct and demonstrates a clear understanding of the vector component concept.</li> <li>- Students can identify the initial and endpoint of vectors and perform picture</li> </ul>	<ul style="list-style-type: none"> <li>- The answer is fully correct and demonstrates a clear understanding of the vector component concept.</li> <li>- The students can explain by setting the initial point and endpoint</li> </ul>	<p>3</p>

representations of vectors  $\vec{w}$  in component form.

- Students can show mathematical expressions and symbols to clarify the point and component of vectors.



- Students can explain the procedures on how to get vector components including the use of mathematics expression and symbolics to get the result of vector  $\vec{w}$  in component form.

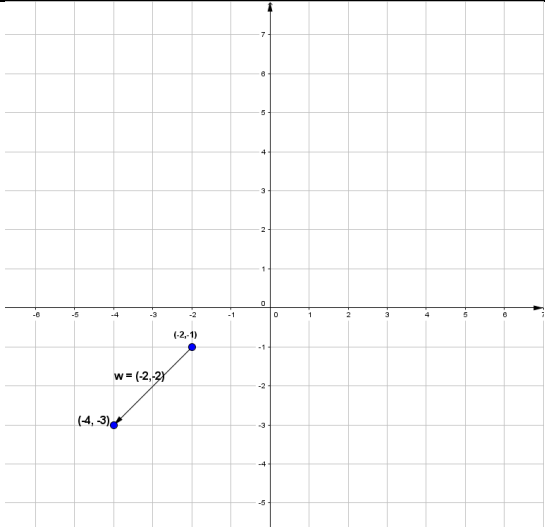
$$\vec{w} = (x_B - x_A, y_B - y_A)$$

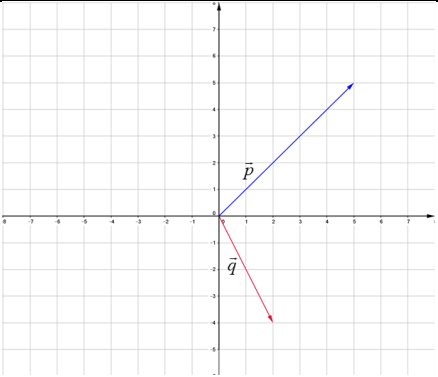
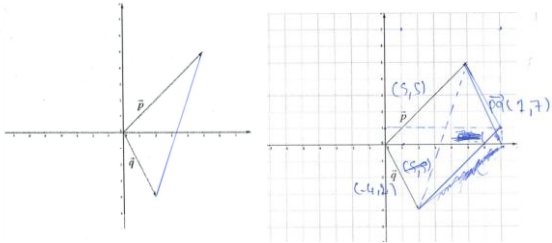
$$\vec{w} = (-4 - (-2), -3 - (1))$$

$$\vec{w} = (-4 + 2, -3 + 1)$$

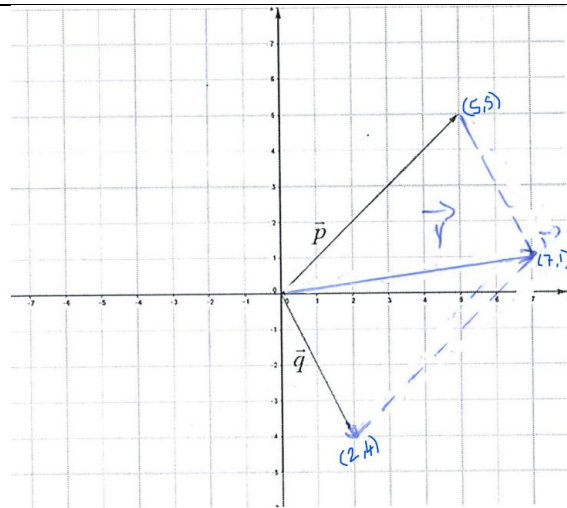
$$\vec{w} = (-2, -2)$$

និយាយពន្យល់ពីរបៀបស្វែងរក  
 ជំនាន់ ក្នុងករណីនេះ គឺជា ចំនុច  
 យើងត្រូវយកគ្រប់គ្រង ជំនាន់  
 គ្រប់គ្រង គឺជា គ្រប់គ្រង គ្រប់គ្រង  
 គ្រប់គ្រង គ្រប់គ្រង គ្រប់គ្រង ។

			<p>The component of vector <math>\vec{w} = (-2, -2)</math> because from the initial point of vector <math>\vec{w}</math> to the endpoint along <math>x'ox</math> decreased 2 units and along <math>y'oy</math> decreased 2 units</p> <p>Another explanation</p> <ul style="list-style-type: none"> <li>- Let <math>A(-2, -1)</math> is an initial point</li> <li>- Let <math>B(-4, -3)</math> is an endpoint</li> </ul> <p>Then</p> $\vec{w} = \overline{AB}$ $= (x_B - x_A, y_B - y_A)$ $= (-4 - (-2), -3 - (-1))$ $= (-2, -2)$ <p><b>Figure:</b></p> <ul style="list-style-type: none"> <li>- Set point initial point <math>(-2, -1)</math> and terminal point <math>(-4, -3)</math></li> <li>- write the vector component <math>\vec{w} = (-2, -2)</math></li> </ul>	
<p>5. Graph <math>\vec{p} + \vec{q}</math> with the same initial point as <math>\vec{p}</math> and <math>\vec{q}</math>. Explain how you get the answer.</p>	No Answer	there is no attempt to answer the question	there is no attempt to answer the question	0
	Incorrect	<ul style="list-style-type: none"> <li>- The answer provided is completely incorrect or does not respond to the given problem.</li> <li>- Students cannot graph vectors <math>\vec{p} + \vec{q}</math> with the same initial point.</li> <li>- Student used symbolic and mathematical expressions in the correct place.</li> </ul>	<ul style="list-style-type: none"> <li>- Students explain in different ways and do not respond to the given problem.</li> <li>- Student can only explain only one or two points, which is not complete the answer.</li> <li>- Students cannot explain by using mathematical expressions or symbols to express calculation procedure.</li> </ul>	1

			<p>         ចូលរកវ៉ិចទ័រផលបូកនៃវ៉ិចទ័រទាំងពីរដោយប្រើប្រាស់ក្របខ័ណ្ឌ          ពោលវ៉ិចទ័រផលបូកនៃ <math>\vec{p} + \vec{q}</math> ជាប់មុំ <math>45^\circ</math> </p> $\vec{p} = (5, 5)$ $\vec{q} = (-4, 2)$ $\vec{p} + \vec{q} = (5, 5) + (-4, 2)$ $= 1(1, 7)$	
<p><b>Objective:</b></p> <p><b>Visual Representation</b></p> <p>To identify the students' abilities to create geometrical drawings to clarify the problem and skills in performing picture representations, including visual, symbolic, and verbal presentations.</p> <p><b>Conceptual Understanding</b></p>	<p>Partly Correct</p>	<ul style="list-style-type: none"> <li>- The answer shows some geometrical drawing in response to a problem given but is incomplete or contains minor errors.</li> <li>- Students can draw the figure <math>\vec{p} + \vec{q}</math> with the same initial point as a diagonal parallelogram but remain missing points such as they drew without using mathematical expressions and symbols.</li> <li>- Students draw a parallelogram which results from the addition of two vectors</li> </ul>	<ul style="list-style-type: none"> <li>- Student can explain the concept of their understanding of addition on vector as the diagonal of a parallelogram.</li> <li>- Student can explain the result but fails to use symbolic or mathematical expressions.</li> <li>- Students can explain the addition of two vectors by using the parallelogram rule without any mathematical procedure and symbols related to visual representation.</li> </ul>	<p>2</p>





फलकानुसार  $\vec{p} + \vec{q}$

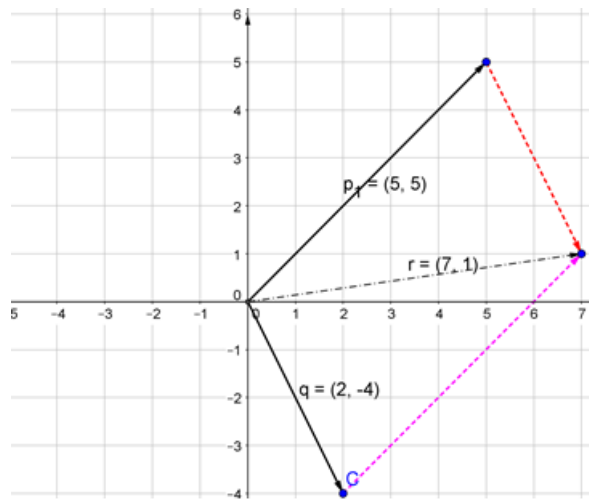
$$\vec{r} = \vec{p} + \vec{q}$$

$$= (5, 5) + (2, -4)$$

$$= (5+2, 5-4)$$

$$\vec{r} = (7, 1)$$

उत्तर:  $\vec{r} (7, 1)$



- Vector  $\vec{p}$  &  $\vec{q}$  has only one initial point **O**
- Draw a parallelogram by graphing vector  $\vec{p}$  &  $\vec{q}$
- Draw a diagonal vector from the initial point **O**
- Use parallelogram's law, we can write  $\vec{p} + \vec{q} = \vec{r}$

**Figure:**

Draw the right figure as instruction below  
 $\vec{p} + \vec{q} = \vec{r}$

## **Appendix 9: Interview Guide for Curriculum Developers**

### **I. Introduction**

The researcher will introduce the research purpose and research objective. The researcher will use guided questions before the primary interview.

### **II. Semi-structure interview**

1. Who ultimately decides the distribution of content for learning?
2. Is there any fixed period of curriculum revision?
3. Before the curriculum development process, did you participate in organizing teams to gather content-related data from schools?
4. Did you integrate some curricula from other countries during the drafting process for curriculum development? If yes, which countries?
5. During the drafting process for curriculum improvement, did you consult or have discussions with experts from other countries?
6. What challenges are you involved in developing the mathematics curriculum?
7. What criteria are used to set the objectives of the mathematics curriculum for each lesson? Specifically, for geometry contents?
8. What challenges does the current mathematics curriculum present to teachers and students in schools?
9. Does the current mathematics curriculum emphasize the integration of technology? If so, how?
10. GeoGebra is a software that can support mathematics teaching and learning. Do you think GeoGebra should be integrated into the mathematics curriculum, particularly for geometry content? If so, how should it be integrated?
11. Do you have any suggestions for improving the mathematics curriculum, particularly in geometry, to achieve better results?

## **Appendix 10: Interview Guide for Teachers**

### **I. Introduction**

The researcher will introduce the research purpose and research objective. The researcher will use guided questions before the primary interview.

### **II. Semi-structure interview**

1. Do you think the mathematics content in the current curriculum is of good quality?
2. Do you know how many years the curriculum and textbooks need to be revised?
3. Have you ever participated in the development of a mathematics curriculum? If so, what role are you responsible for?
4. Does the current mathematics curriculum align with the context of Cambodia, or is it integrated from other countries? If so, which countries?
5. Do you think that the development of a curriculum which integrates the contents from other countries faces challenges in teaching and learning?
6. Do you face any difficulties in teaching and learning mathematics, especially geometry? If so, what are those difficulties?
7. What resources or tools do you use to teach geometry lessons?
8. Did you integrate technology into teaching geometry lessons? If yes, in what way?
9. GeoGebra is a software that can support mathematics teaching and learning. Do you think GeoGebra should be integrated into the mathematics curriculum, particularly for geometry content? If so, how should it be integrated?
10. What suggestions do you have for improving the teaching and learning of geometry to achieve better results?

## **Appendix 11: Interview Guide for Students**

### **I. Introduction**

The researcher will introduce the research purpose and research objective. The researcher will use guided questions before the primary interview.

### **II. Semi-structure interview**

1. What do you think about these problems?
2. How can you draw these pictures?
3. Why do you draw pictures in these ways?
4. Why do you make these mistakes?
5. How do you understand this problem?
6. How do you solve these problems and get these results?
7. How do you think about these test items as a whole after integrating GeoGebra?

## Appendix 12: Research Title

Application Number: HR-ES-000981

### 審査コメント

通知年月日/ Date of notification

2023年5月30日/ May 30, 2023

研究タイトル/ research title

The study on the effectiveness of integrating GeoGebra Software in teaching and learning geometry at upper secondary schools in Cambodia.

申請者名/ name of the applicant

SIENG VEASNA

判定理由又は勧告

## Appendix 13: Research Ethics Clearance

Application Number: HR-ES-000981

### 審査結果通知書/ Research Ethics Clearance

通知年月日/ Date of notification

2023年5月30日/ May 30, 2023

SIENG VEASNA 殿

広島大学大学院人間社会科学研究科長/ Dean, Graduate School of Humanities  
and Social Sciences

小林 信一/ Kobayashi, Shinichi

研究タイトル/ research title

The study on the effectiveness of integrating GeoGebra Software in teaching and learning geometry at upper secondary schools in Cambodia.

申請年月日/ date of application

2023年5月13日/ May 13, 2023

判定結果/ result

承認/ accepted

判定理由又は勧告

別紙参照

**Appendix 14: Approval from MoEYS**



**ព្រះរាជាណាចក្រកម្ពុជា  
ជាតិ សាសនា ព្រះមហាក្សត្រ**

**ក្រសួងអប់រំ យុវជន និងកីឡា**  
លេខ: ៤០១៣ អយក.ខទន

**លិខិតឧទ្ទេសនាម**

**យោង:** លិខិតឧទ្ទេសនាមលេខ៩៥៣ អយក.ខទន ចុះថ្ងៃទី១៦ ខែកុម្ភៈ ឆ្នាំ២០២២ របស់ក្រសួងអប់រំ យុវជន និងកីឡា ។  
- លិខិតចុះថ្ងៃទី២៥ ខែឧសភា ឆ្នាំ២០២៣ របស់ Hiroshima University នៃប្រទេសជប៉ុន ។

លោក **សៀង ខាសនា** គ្រូឧទ្ទេសព័ត៌មានវិទ្យា និងគណិតវិទ្យា បម្រើការនៅវិទ្យាស្ថានជាតិអប់រំ ត្រូវបានអនុញ្ញាតឱ្យចុះធ្វើការសិក្សាស្រាវជ្រាវប្រមូលទិន្នន័យសរសេរនិក្ខេបបទបញ្ចប់ថ្នាក់បណ្ឌិត ចាប់ពីថ្ងៃទី១២ ខែមិថុនា ដល់ថ្ងៃទី២២ ខែកក្កដា ឆ្នាំ២០២៣ នៅវិទ្យាល័យចំណេះទូទៅ និងបច្ចេកទេសហ៊ុន សែន ជម្ពូនី និងសាលារៀនជំនាន់ថ្មី នៃវិទ្យាល័យព្រះស៊ីសុវត្ថិរាជានីក្នុងរាជធានីភ្នំពេញ ។

ថ្ងៃ ២៤ ខែ កញ្ញា ឆ្នាំ ២០២៣  
រាជធានីភ្នំពេញ ថ្ងៃទី ២១ ខែ ឧសភា ឆ្នាំ ២០២៣  
**រដ្ឋមន្ត្រីក្រសួងអប់រំ យុវជន និងកីឡា**



**បណ្ឌិតសភាចារ្យ ហង់ ជួន ណារ៉ុន**

- កន្លែងទទួល:**
- អគ្គនាយកដ្ឋានរដ្ឋបាល និងហិរញ្ញវត្ថុ
  - អគ្គនាយកដ្ឋានឧត្តមសិក្សា
  - អគ្គនាយកដ្ឋានអប់រំ
  - ខុទ្ទកាល័យឯកឧត្តមបណ្ឌិតសភាចារ្យរដ្ឋមន្ត្រី
  - វិទ្យាស្ថានជាតិអប់រំ
  - សាមីជន
  - កាលប្បវត្តិ
  - ឯកសារនា.បុគ្គលិក
- " ដើម្បីជូនប្រជាព័ត៌មាន "
- " ដើម្បីមុខការ "
- " ដើម្បីអនុវត្ត "