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Analyzing Students' Difficulty in Learning Geometry at Upper Secondary School in Cambodia: A Case Study on Vector

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ABSTRACT

This study aims to analyze students' difficulties in learning Geometry at upper secondary schools in Cambodia, focusing on vectors through visual representation and conceptual understanding. The research involved 226 10th-grade students, including 139 females. It used an explanatory sequential mixed-methods design and collected data through tests on vector contents adapted from the Cambodia 10th-grade textbook. The instrument consisted of five test items, each assessing visual representation and conceptual understanding, and semi-structured interview questions. The data analysis employed statistical analysis to see a central tendency of visual representation and conceptual understanding and Wilcoxon Signed Ranks to examine their differences in learning difficulty. The result showed that students solved vector problems related to visual representations better than those requiring conceptual understanding. Students' mean visual representation and conceptual understanding format scores were 6.74 and 4.88, respectively. Wilcoxon Signed Ranks Tests of students' difficulty in solving vector problems, which were significant differences between visual representation and conceptual understanding ($N=226$, $z=-10.76$, $p<.001$, $r=-0.71$), indicating that the difference between students' difficulties in visual representation and conceptual understanding is significant. The study highlighted that (1) students struggled with creating geometrical drawings, including difficulties with visual, symbolic, and verbal representations, and (2) students had difficulty grasping correct mathematical concepts and performing correct mathematical procedures related to visual presentation. The findings suggest focusing on various representations and connecting the meanings of vector operations across these representations to improve students' understanding of Geometry concepts. Additionally, future research should explore visual perception and visual-spatial skills to enhance students' achievement in Geometry.

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INTRODUCTION

Cambodia's education system, particularly general education, has expanded for a number of years after the civil war of 1975-1979 (Khmer Rouge) until the present. The current general education practice only started after the end of this civil war and has been divided into three levels: primary, lower and upper secondary. The Ministry of Education, Youth and Sport (MoEYS) has restructured the education system three times. First, from 1970 to 1986, a ten-year education system was implemented, consisting of four years of primary, three years of lower secondary and three years of upper secondary education (4+3+3). From 1986 to 1994, the education system was changed to eleven years, with five years of primary, three years of lower secondary and three years of upper secondary education (5+3+3). Finally, from 1994 to now, the general education system has been extended to twelve years (6+3+3) to increase teaching-learning hours and enhance student's knowledge (Naron, 2015; SEM & HEM, 2016).

Even though Cambodia's education has increased in the number of years in general education, numerous documents and reports indicate that students' learning achievements are still low compared to ASEAN and OECD (Bhatta et al., 2022; MoEYS, 2018a, 2022, 2023, 2024; UNICEF, 2022). For instance, MoEYS reported that in the Programme for International Students Assessment (PISA), only 7.9% of students had achieved level 2 in reading, and 12% reached level 2 proficiency in Mathematics (MoEYS, 2024). Additionally, the National Learning Assessment (NLA) of November 2021 revealed that students' achievements in both Khmer (the official language of Cambodia) and Mathematics remained low compared to the NLA of 2016 (UNICEF, 2022), especially in Mathematics, where the overall score is only 38% (Bhatta et al., 2022). In Mathematics, the lowest average achievement was in the Geometry domain, among others (Algebra, Statistics, Measurement, and Numbers), with only 35% of sixth graders and 46% of eighth graders students completing the assessment test correctly (Bhatta et al., 2022; MoEYS, 2023). Similarly, the report on the 12th-grade national examination in the academic year 2020–2021 showed that only 47.93% of students could solve Geometry problems correctly (MoEYS, 2022). These indicators reveal insufficient student achievement in learning Geometry.

Geometry is a crucial domain that enhances the visualization of the concepts in school mathematics (Mammarella et al., 2017). Various scholars have found that in learning Geometry, visualization is challenging for students (Baiduri et al., 2020; Dundar et al., 2012; Ramdjid et al., 2022; Yurmalia & Herman, 2021). Moreover, both teachers and students agree that visualization is a complex means for teachers and students to teach and learn Geometry, respectively (Darmofal et al., 2002; Hwang et al., 2007; Perry & Len-Ríos, 2019; Samphantakul & Thinwiangthong, 2019; Utami et al., 2019; van Garderen et al., 2018). Complexities include visualizing 2D and 3D shapes and understanding their geometrics properties, relationships and theorems, which lead to students' difficulties in solving geometric problems.

Gal and Linchevski (2010) found that most students struggle to see and interpret different objects; for example, students have a disability in recognizing and interpreting geometric shapes (circles, squares, triangles), patterns, and figures. This includes figure-ground perception, perception of spatial relationships, visual discrimination, and visual memory in the thinking process required to identify and analyze geometric shapes. This type of visualization can be called visual perception (Gal & Linchevski, 2010; Sari & Slamet, 2018).

Zhang et al. (2014) found that students have difficulty expressing reasoning and describing the relationship between geometric shapes, theorems, and verbal expressions. For example, the students struggle to understand and manipulate geometric figures, mentally manipulating 2D and 3D shapes, such as rotating a shape to view it from angles and understanding transformations, rotations, reflections, and scaling. It is crucial for solving problems that require visualizing spatial relationships and transformations. This type of visualization can be called visual-spatial (Gal & Linchevski, 2010; Zhang et al., 2014).

Žakelj and Klancar (2022) found that students struggle to create geometrical figures to clarify the problems and integrate multiple representations, such as symbolic notation and verbal explanations. It involves creating and using various forms of visual aids to understand and communicate geometric concepts; for instance, students may sketch shapes, angles, and figures to visualize and solve geometric problems, use the coordinate system to plot points, lines, and curves to understand relationships between geometrics entities; employ symbol and notation to represent geometric properties and relationships. This type of visualization can be called visual representation. (Skemp, 1976; Žakelj & Klancar, 2022).

Therefore, there are three main types of visualization that students encounter difficulties in learning Geometry: (1) visual perception, which involves the students' difficulty in seeing and interpreting different objects (Gal & Linchevski, 2010; Sari & Slamet, 2018); (2) visual-spatial skills, which refer to the students' difficulty in expressing reasoning, describing the relationship between geometric shapes, theorems and verbal expressions and creating models (Gal & Linchevski, 2010; Zhang et al., 2014); and (3) visual representation, which involves the students' difficulty in creating geometrical figures to clarify the problem and integrate multiple presentations (Skemp, 1976; Žakelj & Klancar, 2022). The figure below shows the students' difficulty in learning Geometry in relation to visualization.

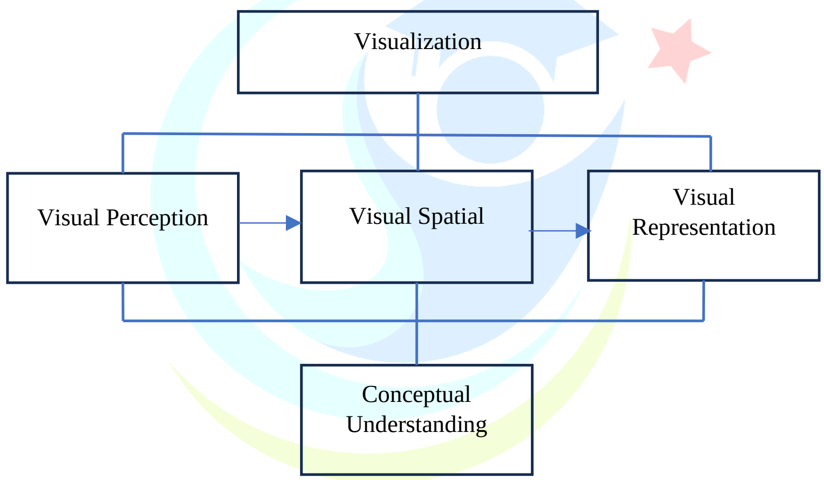


Figure 1. students' difficulty in learning geometry through visualisation

Sources: The researcher developed based on the literature review

Figure 1 shows students' difficulties in learning Geometry through visualization, consisting of three interrelated features: visual perception, visual-spatial, and visual representation. These relations mean that to solve a geometric problem, a student might first use visual perception to identify the shapes involved, then apply visual-spatial skills to manipulate these shapes mentally, and finally use visual representation to draw diagrams or graphs that aid in solving the problems (Gal & Linchevski, 2010; Jones & Tzekaki, 2016b).

Among the three types of visualization, different education stages follow the implementation of visualization in general education. In primary education, the focus is on visual perception; this leads to visual-spatial skills in lower secondary education, and finally, the emphasis shifts to visual representation in upper secondary education. Therefore, understanding how visualization works together in Geometry can lead to more effective teaching techniques, better learning success, and a deeper understanding of the subject, which can help the students develop a conceptual understanding (Gal & Linchevski, 2010; Jones & Tzekaki, 2016a; Khalil et al., 2024; Lowrie et al., 2019; Majeed & ALRikabi, 2022; Porat & Ceobanu, 2024).

According to the national examination report for 12th grade in the physical year 2020-2021, Cambodian students' achievement was still below average in vector problem-solving abilities; 47.93% of students could solve vector problems correctly (MoEYS, 2022). Based on the previous literature, visual representation and conceptual understanding can be stated as the difficulties in solving vector problems for students in upper secondary education. Therefore, the topic of vectors was chosen for constructing the test items to analyze the students' difficulty in learning Geometry through visual representation and conceptual understanding in upper secondary school in Cambodia. There are two reasons for selecting the vector topic: (1) vector forms the foundation concept of Geometry, which is related to many advanced topics in mathematics, such as linear algebra, physics, engineering, and navigation. Students need to understand vectors to grasp more complex concepts in their studies (Dray & Manogue, 2023), (2) vector is a foundation topic addressed in Cambodia's first grade (10th grade) of the upper secondary school Mathematics curriculum (MoEYS, 2018b, 2018c) and textbook (MoEYS, 2020a, 2020b). Students who understand vector concepts can help them understand other related topics more quickly.

Through the discussion above, it is clear that students with a poor understanding of visual representation can lead to poor conceptual understanding. While this study mainly focuses on upper secondary education, visual representation and conceptual understanding are the focal targets. Thus, the test items may include visual perception and visual-spatial. Even though these are not directly addressed, they are implicitly considered because they are interrelated. Hence, this study considers the other two through the analysis of test items because they are embedded in the visual representation.

In this sense, this current study aims to analyze the students' difficulty in learning Geometry through visual representation and conceptual understanding of vector concepts. Visual representation refers to students' ability to create geometrical drawings to clarify the problem and skills in performing picture representations, including visual, symbolic, and verbal presentations. Conceptual understanding refers to students' ability to grasp mathematical concepts and perform a correct mathematical procedure in relation to visual presentation. Two main questions have been set for this study:

1. what are the students' difficulties in learning Geometry through visualization?
2. What are students' difficulties in solving vector concepts that differ between visual representation and conceptual understanding?

METHODOLOGY (12 Calibri)

Research Design

The study used an explanatory sequential mixed-method design because this research design is well-suited for a comprehensive identification of students' difficulties in learning Geometry by integrating quantitative and qualitative data. This approach allows for validation and triangulation of findings, complements the strengths of both data types, facilitates theory building, and provides practical implications for educational settings (Creswell & Creswell, 2018; Gay et al., 2012).

Participants

The participants of this study were selected from two high schools in Phnom Penh, the capital city of Cambodia. The students were chosen from 10^{th} grade during the physical year 2022-2023. The selection of students based on school principal designation followed a convenience sampling method, where all accessible students at the selected schools were invited to participate in this study while the test was being administered. The selection procedure was based on the class designed by the school principal, which was not intended to create a representative sample of the Cambodian high school student population. The participants of this study were 226 high school students, including 139 females, who were asked to answer the test. Twelve students were selected for interviews based on the result of scores, low, medium and high to triangulate the test results.

Instrument

The test items were developed by the researcher and adapted to contents and sub-contents of vector concepts based on the Cambodia Mathematics curriculum for upper secondary school published in 2018 (MoEYS, 2018b, 2018c) and the textbook published in 2020 (MoEYS, 2020b) to assess students' difficulties in learning Geometry, particularly visual representation and conceptual understanding. The test items consist of five open-ended questions covering the content of vector and vector operations followed by three sub-contents named (1) Meaning of a Vector, (2) Addition and Subtraction of Vectors, and (3) Component of a Vector in a Plane. Each of the five test items includes both a visual presentation and conceptual understanding. The researcher developed the draft test items and piloted them twice before distributing them to the target students to answer the test.

Three experts were consulted in developing the test items. Two teachers familiar with vector concepts from their experiences in teaching 10th-grade and another expert from the National Institute of Education (NIE) of Cambodia majoring in mathematics checked the validity of the test items to ensure whether they meet the current context. The reliability of the first pilot test items, measured by Cronbach Alpha, was 0.786; however, the allotted time of one hour (50 minutes in teaching hour) was insufficient for the students to solve all the problems.

After the first pilot, the author discussed the test items with experts. Then, item 1 was revised and split into two questions following discussions with the teachers and a mathematics expert, as it originally contained two distinct questions. Item 5 was removed because its content was beyond the scope of 10th-grade students. The second pilot was conducted to confirm if the revised time limit was acceptable. The reliability statistics, confirmed with a Cronbach's Alpha of 0.904, proved that the duration of 50 minutes was suitable for implementing the test, and it confirmed that the test items could be measured. The development of test items arranged by each sub-content is shown in Table 1.

Table 1. The development of test items arranged by each sub-content

Sub-Content	**Test Items of Visual Representation**	**Test Items of Conceptual Understanding**
Meaning of Vector	1, 2	1, 2
Addition and Subtraction of Vector	3, 4	3, 4
Component of a Vector in a Plane	5	5
Total number of test items	**5**	**5**

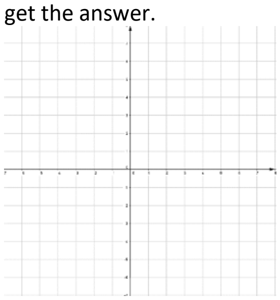
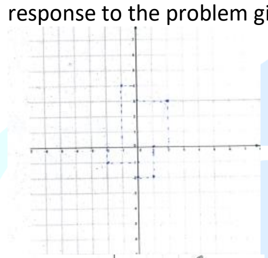
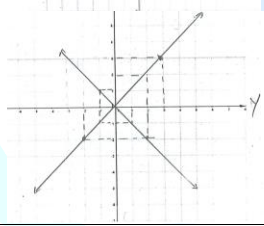
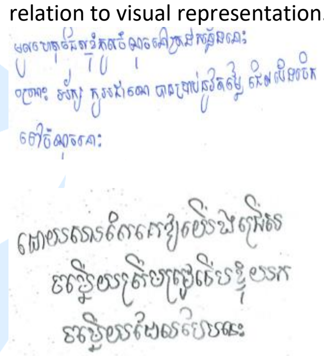
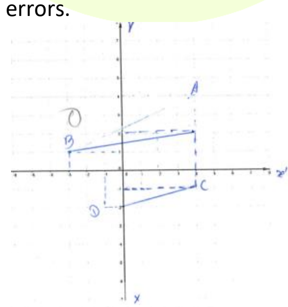
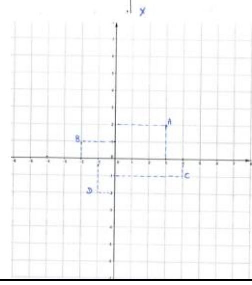
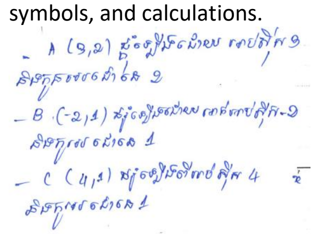
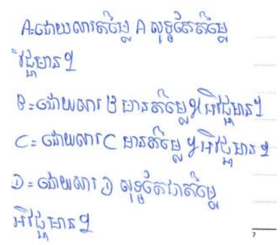
Source. The researcher developed by adapting the Cambodia textbook in 10th- grade (MoEYS, 2020b)

Table 1 shows two test items covering the sub-content of the Meaning of Vector, another two test items covering the sub-content of Addition and Subtraction of Vectors, and one test item dealing with the sub-content of Component of a Vector in a Plane. Both visual representations and conceptual understanding are encompassed in each of these test items.

Data Analysis

To analyze the results, the researcher created a scoring rubric and marked the student responses after consulting with experts and through peer review, which included lab members, two 10th-grade teachers, and one expert from NIE. The rubric categories that emerged are (1) no response, where there is no attempt to answer the question mark as zero; (2) incorrect, where the answer provided is completely incorrect mark as one; (3) partly correct, where the answer shows some understanding of the concept but is incomplete or contains minor errors mark as two; and (4) correct, where the answer is fully correct and demonstrates a clear understanding of the concept mark as four. An example of a scoring rubric is shown in Table 2.

Table 2. The Scoring Rubric of Test Item Number 1

Test Items and Objective	Response	Visual Representation	Conceptual Understanding	Scoring
1. Set four points which are given A(3,2), B(-2,1), C(4,-1) and D(-1,-2) in the figure below and explain how you get the answer.  **Objective:** **Visual Representation** To identify the students' abilities to create geometrical drawings to clarify the problem and skills in performing picture representations, including visual, symbolic, and verbal presentations. **Conceptual Understanding** To describe the students' abilities to grasp mathematical concepts and perform correct mathematical procedures in relation to visual representation.	No answer	There is no attempt to answer the question	There is no attempt to answer the question	0
	Incorrect	The answer provided is completely incorrect in response to the problem given.  	The answer provided is completely incorrect in performing the mathematical procedure in relation to visual representation. 	1
	Partly correct	The answer shows some parts of drawing the pictures but is incomplete or contains minor errors.  	The answer shows some parts of the correct procedure using mathematical expressions, symbols, and calculations.  	
	Correct	The answer is fully correct and demonstrates a clear understanding of the concept of the problem by plotting four	The answer is fully correct and demonstrates a clear understanding of the concept of the problem using mathematics	

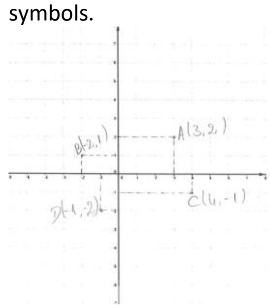
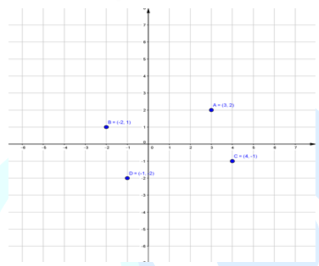
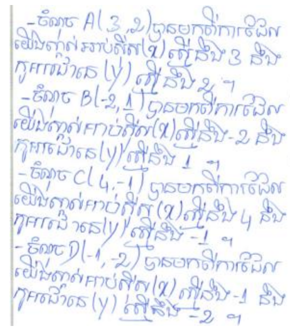
	points at the proper position and performing picture representation using mathematical expressions and symbols.  	expression and symbols related to visual representation.  By the given problem, we can set the points as below: - Point $A(3,2)$ at the first quadrant because $x_A = 3 > 0$ and $y_A = 2 > 0$ - Point $B(-2,1)$ at the second quadrant because $x_B = -2 < 0$ and $y_B = 1 > 0$ - Point $C(-4,-1)$ at the third quadrant because $x_C = 4 > 0$ and $y_C = -1 < 0$ - Point $D(-1,-2)$ at the fourth quadrant because $x_D = -1 < 0$ and $y_D = -2 < 0$ Figure: - Set the four points at the right position

Table 2 shows that each question consisted of the student responses in both visual representation and conceptual understanding. The student who responded correctly achieved three scores for each test item. The total scores of five test items were marked 30, with 15 for visual representation and another 15 for conceptual understanding.

The data analysis drew from two types of student responses: quantitative and qualitative. The quantitative data were obtained from students' scores, following a scoring rubric. All the scores were input into an Excel file and cleaned before import to the tool. IBM SPSS Statistics Data Editor version 27 was used as a tool to analyze the quantitative data. Data analysis was carried out to describe the students' difficulties in solving vector problems related to visual representation and conceptual understanding. It was accomplished by determining descriptive statistics, namely the measurement of central tendency and the size of data distribution.

Furthermore, the difference between the mean scores in students' visual representation and conceptual understanding was compared using the Wilcoxon Signed Ranks nonparametric because some of the skewness values for two data groups out of the interval -1 to 1, it can be said that the two data groups are not normally distributed (Morgan et al., 2011).

Moreover, qualitative data analysis was obtained from students' responses and semi-structured interviews to triangulate the results corresponding to visual representation and conceptual understanding. All the interview results were coded following a scoring rubric for three types of student achievement: low, medium, and high. The analysis focuses on two main points:

1. The researcher identified the students' difficulties in creating geometrical drawings to clarify the problem of visual representation and students' ability to grasp the mathematical concept of conceptual understanding.

   In this step, the researcher first collected the students' responses from each test item. All test items were arranged into the appropriate responses and examined using a scoring rubric (no response, incorrect, partly incorrect, and correct) to determine whether the students drew the figures corresponding to the given problem.

Second, the researcher collected the students' responses from each test item and assessed students' understanding of various vector problems of increasing complexity. This assessment is done by observing how they approach and solve these problems, noting their understanding of vector operations (addition, subtraction, scalar multiplication) and how to find the vector components forms. It also included interpreting information from visual representation to mathematical expression.

2. The researcher analysed the students' skills in performing picture representation, including visual, symbolic, and verbal presentation of visual representation and analysed the students' ability to perform correct mathematical procedures in relation to visual representations of the conceptual understanding.

   This step first identified the students' drawing skills corresponding to picture representation and whether the relation of picture representation responds to the given problem, including presenting symbolic and verbal presentation in the correct order of the given problems.

   Second, the researcher evaluated how well students understand the relationship between the visual data and the mathematical calculations required. It includes assessing students' calculations based on the visual representation provided, determining whether they use the correct formulas, applying appropriate mathematical procedures and arriving at the correct result.

Finally, the researcher examined the relationship between visual representation and conceptual understanding to see whether the two correspond and support each other in solving Geometry problems.

Research Ethics

The research title, data collection method, and participant sampling were presented to and approved by the Graduate School Ethics Committee prior to data collection. Regarding confidentiality and anonymity, identity coding was conducted for each interviewee (i.e., CPF1ST1, CPF1ST2, CPI1ST1, CPI2ST2,…). The tests were conducted at the target schools with a permission of school principal, and participants. Informed consent was obtained from each participant; the form indicated the purpose of the study and asked permission to record and use the data. Participants were asked to choose a convenient time for the interview. The respondents had the right to skip any questions at any time during the interviews.

RESULTS

The analysis of the students' responses obtained the following results:

Students' difficulty in learning Geometry through visual representation and conceptual understanding in solving vector concept problems

The result of students' responses to each test item for visual representation and conceptual understanding of vector concepts, in general, can be indicated by their responses from the scores obtained after taking the test, along with the scoring rubric. The results from the descriptive statistics data on visual representation and conceptual representation in solving vector concept problems are shown in Table 3.

Table 3: Descriptive Statistic Data

Description	N	Mean	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Statistic	Std. Error
Visual Representation	226	6.74	2.964	0.938	0.162

Conceptual Understanding	226	4.88	3.419	0.628	0.162
Valid N (listwise)	226				

Table 3 shows that, based on the mean scores of the students' responses, their ability to answer visual representation problems is better than their conceptual understanding. In the visual representation of students' responses, the mean score was 6.74, while for conceptual understanding, it was only 4.88. Here, it can be seen that drawing figures in a visual representation format is more accessible for students because it only requires understanding the problem given, including symbolic and verbal presentation, unlike the calculation procedures such as addition, subtraction, and scalar multiplication of vector problems in conceptual understanding are often challenging to grasp and require more conceptual knowledge to strengthen their interpretation of visual representations into mathematical procedures. However, they are below medium.

Therefore, this result indicates that students struggle with visual representations and conceptual understanding in problem-solving in Geometry, particularly with vector concepts. The percentage of students who answered correctly on each test item is shown in Figure 2.

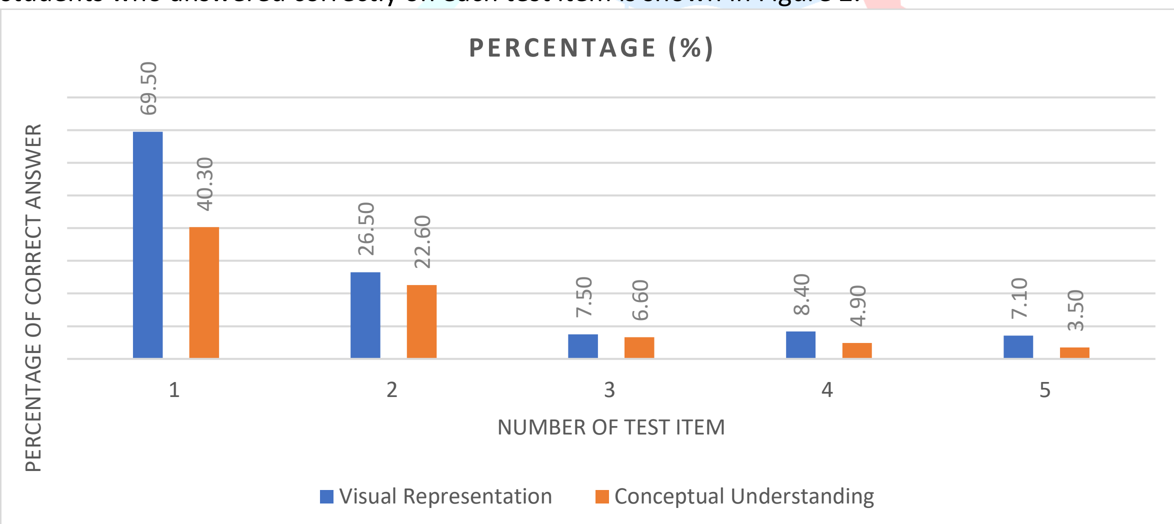
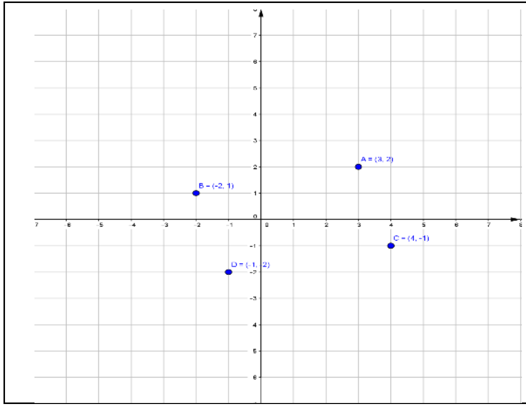


Figure 2: Percentage of students who responded correctly for each test item

Figure 2 shows that the percentage of students who answered test items correctly for visual representation was higher than for conceptual understanding, respectively, based on the context of the problems. The most significant difference is test item 1, which is 29.20%. The figure below shows test item number 1 and its expected answer of visual representation and conceptual understanding.

Test Item 1. Set four points which are given A(3,2), B(-2,1), C(4,-1) and D(-1,-2) in the below figure and explain how you get the answer.

Visual Representation	**Conceptual Understanding**



By the given problem, we can set the points as below:

- Point $A(3,2)$ at the first quadrant because $x_A = 3 > 0$ and $y_A = 2 > 0$
- Point $B(-2,1)$ at the second quadrant because $x_B = -2 < 0$ and $y_B = 1 > 0$
- Point $C(4,-1)$ at the third quadrant because $x_C = 4 > 0$ and $y_C = -1 < 0$
- Point $D(-1,-2)$ at the fourth quadrant because $x_D = -1 < 0$ and $y_D = -2 < 0$

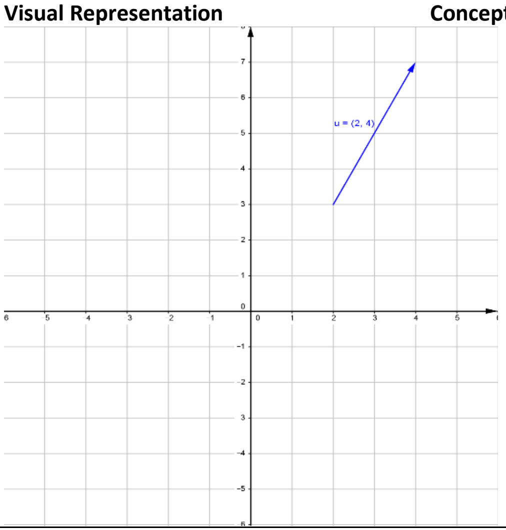
Figure 3: Test item Number 1 and expected answer from respondents

Figure 3 shows the approach to plot these four points correctly. Students must first have the skills to plot four points labelled $A, B, C,$ and D on a Cartesian plane, each with its corresponding coordinates. Second, students need the skills to place those labels at the right positions, such as point A is located at $(3,2)$, point B is located at $(-2,1)$, point C is located at $(4,-1)$, and point D is located at $(-1,-2)$. This visual representation helps to identify the positions in relation to the x and y axes distinctly. They need to know that an ordered pair (x, y) describes the place of a point on the coordinate plane. The first coordinate of x tells students the position of a point along the x-axis, and the second coordinate of y tells the position of a point along the y-axis. Then, to plot a point $A(3, 2)$, students need to start at $(0,0)$, which is the point where the axes of (x, y) meet. Move three units to the right and two units up to reach the point $(3, 2)$. Apply this method to the other points to get $A(3,2), B(-2,1), C(4,-1)$ and $D(-1,-2)$. Following this mathematical procedure, students faced difficulties in identifying the construction of the coordinates of points.

On the other hand, in test item 3, students who answered the test item correctly in visual representation were almost similar to those in conceptual understanding. The significant difference of test item 3 is 0.9%. The figure below shows test item number 3 and its expected answer.

Test Item 3. Draw the vector with an initial point (2,3), then find the terminal point by giving a vector in component form $\vec{u} = (2,4)$ and explain how you get such an answer by showing the graph.

Visual Representation



Conceptual Understanding

- Set the initial point $A(2,3)$ and the endpoint (x_B, y_B)
- Then write a sentence $\overrightarrow{AB} = \vec{u}$
- we obtained, $(x_B - 2, y_B - 3) = (2,4)$

$$\Rightarrow \begin{cases} x_B - 2 = 2 \\ y_B - 3 = 4 \end{cases} \iff \begin{cases} x_B = 4 \\ y_B = 7 \end{cases}$$

Therefore $B(4,7)$ is the end point of vector $\vec{u}$

Figure 4: Test item 3 and expected answer from respondents

Figure 4 shows that students should have the skills in vector operation to find the terminal point by giving a vector component form of $\vec{u}=(2,4)$ and an initial point of $(2,3)$. Given a vector component form of $\vec{u}=(2,4)$, this means that the coordinate moves to the right by two units along the $x-axis$ and four units along the $y-axis$, resulting in the terminal point (4,7). Then, they need the skills to draw a vector, which direction from the initial point to the terminal point. This visual representation can help students draw the vector and use symbolic notation to perform the vector representations.

However, students' conceptual understanding faces more difficulty in solving vector operations because this problem is abstract and requires critical thinking skills. To solve this problem, students need to interpret from picture representation to mathematical expression corresponding to visual representation. For example, plot a point (2,3), labelled as A and use the symbolic presentation to determine the terminal point as $B(x_B, y_B)$. Then, they should write a mathematical expression $\overrightarrow{AB}=\vec{u}$. By processing the vector operation, we obtained the following $(x_B-2, y_B-3)=(2,4)$. By solving this system of equations, we find $x_B=4$ and $y_B=7$. Therefore, $B(4,7)$ is the terminal point of $\vec{u}$. Following this procedure, students had difficulty in interpreting visual representations to perform correct mathematical procedures. The students' abilities to answer this test item were almost similar in solving problems related to visual representation and conceptual understanding. Their achievement scores were below average, differing by only 0.9%. It was indicated that students' low ability to understand visual representation can lead to low achievement in conceptual understanding.

Students' difficulties in solving vector concepts differ between visual representation and conceptual understanding.

Although there are appearance differences in students' difficulty in solving vector problems in visual representation and conceptual understanding, these differences need to be tested statistically. The results of the Wilcoxon signed ranks test are shown in Table 4.

Table 4. The differentiation between visual representation and conceptual understanding

Ranks

		N	**Mean Rank**	**Sum of Ranks**	**z**	**r**	**Sig.**
Conceptual Understanding - Visual Representation	Negative Ranks	164[a]	96.99	15906.50	-10.725	-0.7132	p<.001
	Positive Ranks	18[b]	41.47	746.50			
	Ties	44[c]					
	Total	226					

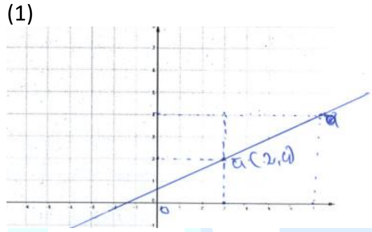
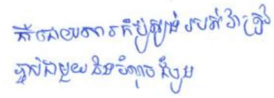
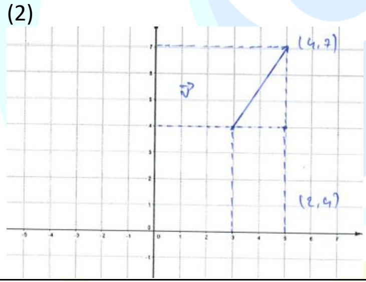
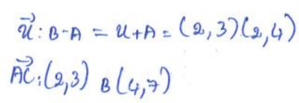
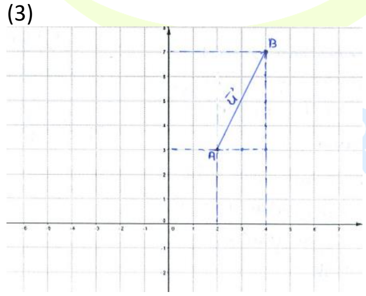
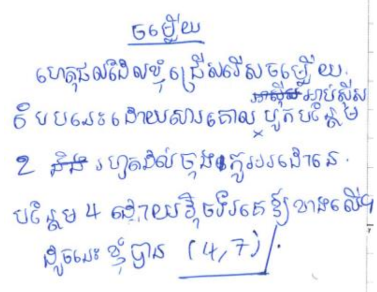
Table 4 shows the result of the Wilcoxon signed ranks tests of students' difficulty in solving vector problems, which are significant differences between visual representation and conceptual understanding (N=226, z=-10.76, p<.001, r=-0.71).

There are 164 achieved conceptual understanding scores lower than the visual representation scores. The mean rank is 96.99; 18 students who achieved conceptual understanding scores were higher than the visual representation scores, with the mean rank being 41.47; and 44 students who achieved conceptual understanding and visual representation scores were equal, a large effect size according to Cohen (1988).

This result shows that students' understanding is still not deep and still depends on the context of the problems. Therefore, students need to strengthen visual representation and conceptual understanding skills to improve their abilities in solving geometric problems, particularly vector problems. Moreover, the analysis of test items and semi-structured interviews with students corresponding to visual representation and conceptual understanding revealed the following:

1. Students have difficulty creating geometrical drawings to clarify the problem and grasp mathematical concepts because they are poor in understanding the foundation concept of vector construction. Here is an example of students' responses to test item number 3, as shown in Table 5.

Table 5. The example of students' response for test item number 3

Test Item	**Visual Representation**	**Conceptual Understanding**
3. Draw the vector with an initial point (2,3), then find the terminal point by giving a vector in component form $\vec{u} = (2,4)$ and explain how you get such an answer by showing the graph.	(1) 	 *This is because its components are connected to the initial point.*
	(2) 	
	(3) 	 *I chose this answer because I added two units to the x-axis and four units to coordinate using the vector given above. Therefore, I obtained (4, 7).*

Comprehensive analysis shows that students who had difficulty answering test items revealed specific characteristics based on their responses. These characteristics included partly correct answers, indicating that some students can solve parts of the problems related to vector contents, but their solutions were incomplete or contained minor errors; incorrect answers indicated students who provided the answer completely incorrect; and no response indicated students tended not to understand the problem and did not attempt to answer the test items. For example, in Table 4, students might struggle to understand what the vector component $\vec{u} = (2,4)$ means in terms of movement along the x-axis and y-axis; accurately plotting the initial point (2,3) on the graph can be challenging; misinterpreting the scale or units on the graph can lead to incorrect placement; drawing a straight line or arrow from the initial point to the terminal point is not clearly differentiate; unable to clarify the direction and length of the vector because they must accurately reflect the given components; unproperly labelling the initial point, terminal point, and vector on the graph because it is crucial for clarity; unfamiliar with the coordinate system, including positive and negative direction along the axes. Thus, these difficulties can make it challenging for students to grasp their understanding of vector representation and pointing on graphs.

2. Students have difficulty performing picture representation, including visual, symbolic and verbal presentation and performing correct mathematical procedures in relation to visual representation. For example, in Table 5, students have difficulty interpreting pictures, visualizing vector construction and their properties, struggling to see the connection between vector components, initial and terminal points, and translating word problems into visual formats. They also have trouble understanding and using mathematical symbols correctly, explaining mathematical concepts verbally and translating verbal descriptions into mathematical expressions. Additionally, students have difficulty applying the correct mathematical procedure to solve problems involving vector concepts and problem identifying and using the correct formulas based on the visual representation.

Semi-structured interviews were conducted after analyzing students' answers in order to triangulate answers and explore students' understanding in finding solutions to geometrical issues using the students' answer sheets as a guide. The interviews were conducted with twelve students, four of whom represented low achievement groups (CPL), four of whom represented medium achievement groups (CPM) and four of whom represented high achievement (CPH).

The summaries of what was said in the interviews about the difficulty in solving vector problems by test items are shown in Table 6.

Table 6. Summary of the interview results

Participants' Code	**Interview summary**
CPL	- Students cannot distinguish between what they know (data) and what they do not know (questions), so they do not understand the questions well and tend only to rewrite given problems. - Students have been unable to determine whether the information on the given problem is adequate. -Students have been unable to identify what conditions must be met to solve the problem. -Students attempt to comprehend and restate the problem in a more operational (solvable) form. - Students attempt to devise or consider a problem-solving strategy.
CPM	- Students thoroughly understand the problem and can devise problem-solving procedures or contain minor errors. - Students are able to solve problems according to given problems, but they are incomplete. - Students understand the problem well and can devise problem-solving procedures, but some replacement signs are missing during the calculation procedure or when substituting the formula. - Students missed writing some parts of symbols and verbal expressions because they thought using them was unnecessary.
CPH	- Students understand the problems well, draw geometrical figures, devise problem-solving procedures, and solve problems according to the given problems. - Students re-examined the procedure and the completion results. - Students have a good understanding of the problems to create geometrical drawings to clarify the given problem, skills in performing picture representations, ability to grasp mathematical concepts, and ability to perform a correct mathematical procedure in relation to visual representation.

According to the data presented in Table 6, it is clear that students' difficulties in solving Geometry problems are due to a poor understanding of visual representation and conceptual understanding of the problem involving Geometry, particularly in vector content.

DISCUSSIONS

The result of this study found that students' difficulty learning Geometry is due to a poor understanding of visualization, particularly visual representation, which can lead to a poor conceptual understanding in upper secondary school. Results of the test items show that students were unable to create geometrical drawings to clarify the problem and provided their skills in performing picture representation. These make the students unable to grasp mathematical concepts and perform correct mathematical procedures. This indicates that students have difficulty solving Geometry problems due to their lack of understanding of visual representation, which can lead to a lack of conceptual understanding.

Similarly, Žakelj and Klancar (2022) studied the role of visual representation in Geometry learning. They found that students' difficulties in solving Geometry problems might stem from challenges in visualizing geometric concepts and understanding the relationships between geometric objects. Addressing these issues through effective teaching strategies and integrating visual representations can help improve students' conceptual understanding and problem-solving abilities in Geometry.

Likely, Utami et al. (2019) studied the identification of the visual representation ability of junior high school students in solving Geometry problems and also found that students with low visual representation abilities often face difficulty in representing questions on the drawings. These students are not given the opportunities to present their own ideas, making it difficult for them to answer questions that differ from the problem given by teachers. This means that the students require more critical thinking skills.

While visual perception and visual-spatial skills are also important in solving Geometry problems, emphasizing their relation with visual representation is crucial for fostering deep understanding, more effective teaching techniques, better learning success, and a deeper understanding of the subject, coherent learning progression and trajectory which can improve students develop a conceptual understanding (Gal & Linchevski, 2010; Jones & Tzekaki, 2016b; Khalil et al., 2024; Lowrie et al., 2019; Majeed & ALRikabi, 2022; Porat & Ceobanu, 2024). This indicated that students need to build their foundation from lower grades, which is necessary to strengthen conceptual understanding.

These findings certainly have implications for the learning trajectory that must be carried out when teaching vector topics and other Geometry content in the classroom. However, students need to learn more about interpreting vectors and geometrical content in various representations and develop a deeper conceptual understanding of Geometry. We also need to consider the importance of learning representation in vector topics. This is because students' understanding still depends on the visual representation of questions, which can lead to improved conceptual understanding. As vectors are one of the foundation topics in the Geometry domain and are useful for physics, architecture, and navigation, the teacher must ensure that students understand the concept of vectors (Wutchana & Emarat, 2011).

CONCLUSIONS

The analysis of students' difficulty in learning geometry at upper secondary school in Cambodia with a focus on vector contents consisting of visual representation and conceptual understanding. Among those, students have difficulty creating geometrical drawings to clarify the problem and grasp mathematical concepts because they are poor in understanding the foundation concept of vector construction, as well as difficulty performing picture representation, including visual, symbolic and verbal presentation and performing correct mathematical procedures in relation to visual representation.

However, the analysis results revealed that students' ability to answer visual representation questions is better than their conceptual understanding, but both tend to be low. Wilcoxon Signed Ranks' results indicate that the ability of students to understand vector concepts with visual representation and conceptual understanding is significantly different. On the other hand, the student results are below average; thus, students need to strengthen their visual representation and conceptual understanding skills to improve their abilities in solving geometric problems, particularly vector problems.

According to this analysis, Cambodian upper secondary school students must strengthen their visual representation and conceptual understanding knowledge to improve learning achievement, including enhancing teacher training, improving curriculum design, and integrating interactive learning tools to address these difficulties and improve overall mathematical proficiency.

While this research mainly focuses on the analysis of visual representation and conceptual understanding, further research should be done on visual perception and visual-spatial skills. Analysis of all these visualizations will contribute to the holistic improvement of students' difficulty, and Cambodian students can better achieve and promote understanding of Geometry concepts.

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